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AUTOMORPHISMS OF L^*-ALGEBRAS*

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In this paper we are concerned with some properties of (algebraic)*-automorphisms and *-isomorphisms of semi-simple L^*-algebras. As a consequence of the inner product uniqueness theorem for L^*-algebras established earlier ([4], see Theorem 1 below), it follows that every *-isomorphism φ of a semi-simple L^*-algebra L is necessarily topological and moreover φ is a semi-L^*-isomorphism if L is simple (Corollary to Theorem 1). From these results we deduce that a *-isomorphism of a semi-simple L^*-algebra can be expressed in terms of partial semi-L^*-isomorphisms (Theorem 2).

We give some conditions under which a *-automorphism is automatically unitary. While a *-automorphism of any finite-dimensional simple L^*-algebra is unitary (Corollary to Proposition 2), this result holds for an infinite-dimensional simple L^*-algebra provided it is of classical type (Theorem 3). Under additional conditions on the automorphism, the same result holds also for the general simple L^*-algebra (see §2). Actually, it is our conjecture that the result is valid even without the additional conditions.

We introduce a notion of regularity for automorphisms of semi-simple L^*-algebras and show by means of a category argument that such automorphisms exist whenever the L^*-algebras are separable (Theorem 4). For automorphisms which are inner, a criterion for regularity is obtained (Proposition 7) which coincides with the one given by Gantmacher for the regularity of automorphisms of semisimple Lie algebras.

1. Preliminaries and structure of *-isomorphisms. Let L be a real or complex Lie algebra of arbitrary dimension. L is called an L^*-algebra if (i) L is equipped with an inner product relative to which it is a Hilbert space; (ii) L is closed for a *-operation $x \to x^*$ which satisfies the connecting relation

$$<[x,y],z> = <y,[x^*,z]> ,$$

where $[\cdot]$ as usual stands for the Lie bracket.

If the centre of L (as a Lie algebra) is zero, L is called semi-simple. L is called simple if it is of dimension greater than one and contains no closed ideals other than $\{0\}$ and L.

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