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PRODUCTS IN SHEAF-COHOMOLOGY

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Introduction. The natural setting for a theory of sheaves is a *site*, i.e. a category C topologized in the sense of Grothendieck (cf. VERDIER [1963]). We shall consider a sheaf A of rings on a site C and the category $\mathcal{A}$ of sheaves of A-modules. When we speak of sheaf-cohomology, we shall mean the right-derived functor of $\Gamma = \mathrm{Hom}_{\mathcal{A}}(A, -)$, that is: $\mathrm{Ext}_{\mathcal{A}}(A, -)$.

In $\mathcal{A}$ one has a tensor-product and a local Hom (denoted: $\mathcal{H}om$) with the familiar exactness properties and the usual adjointness. Our problem is to associate to a pairing

$$F \otimes F' \to G \tag{1}$$

or, equivalently, to a morphism

$$F \to \mathcal{H}om(F', G) \tag{2}$$

of objects of $\mathcal{A}$ a canonical cohomology product

$$H^p(F) \otimes H^q(F') \to H^{p+q}(G) \tag{3}$$

which respects coboundary operators in the usual way. Universality of the cohomology functor does not help, since $H^p(F \otimes -)$ does not, in general, yield a connected sequence of functors on any useful subcategory of short exact sequences. We shall exhibit two constructions for a product (3) which arise in different natural habitats but coincide in the context described above.

An obvious thing to try is the conversion of the first factor $H^p(F)$ into $\mathrm{Ext}^p(F', G)$ via (2) followed by the application of the ever-available Yoneda-product

$$\mathrm{Ext}^p(F', G) \times H^q(F') \to H^{p+q}(G).$$

More precisely, the transition from $H^p(F)$ to $\mathrm{Ext}^p(F', G)$ is accomplished by applying H^p to (2) and then using the edge-morphism