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ON A RIEMANNIAN SPACE ADMITTING MORE THAN ONE SASAKIAN STRUCTURES

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Introduction. Let M^n be a connected n-dimensional Riemannian space. A unit Killing vector field ξ^h is called a Sasakian structure if it satisfies

$$\nabla_j \nabla_i \xi^h = \xi_i \delta_j^h - \xi^h g_{ji},$$

where g_{ji} is the Riemannian metric and ∇_j means the Levi-Civita covariant differentiation[1]. Recently Y. Y. Kuo proved that if M^n admits two Sasakian structures orthogonal to each other it admits one more Sasakian structure orthogonal to them[2].

Our interest and purpose of this paper are to study M^n admitting (i) $r(>3)$ Sasakian structures orthogonal to one another and (ii) 2 Sasakian structures not orthogonal to each other.

1. Preliminaries. Consider a Riemannian space M^n with a Sasakian structure ξ^h. If we put $\varphi_i{}^h = \nabla_i \xi^h$, the following relations hold good:

$$(1)\qquad \begin{aligned} &\xi^r \xi_r = 1, \qquad \varphi_i{}^r \xi_r = 0, \qquad \xi^r \varphi_r{}^h = 0, \\ &\varphi_{ji} \equiv \varphi_j{}^r g_{ri} = -\varphi_{ij}, \\ &\varphi_i{}^r \varphi_r{}^h = -\delta_i^h + \xi_i \xi^h, \\ &\nabla_j \varphi_i{}^h = \xi_i g_{jh} - \xi^h g_{ji}. \end{aligned}$$

Let $M^{n+1} = M^n \times R$ be the product manifold of M^n with a line, and define a tensor Φ of type (1, 1) by

$$\Phi = \begin{pmatrix} \varphi_i{}^h & -\xi^h \\ \xi_i & 0 \end{pmatrix},$$

1) We follow the notations in [5] and [6].
2) Y. Y. Kuo, [1].