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REMARK ON BEHAVIOR OF SOLUTIONS OF SOME PARABOLIC EQUATIONS

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1. Consider a parabolic equation

$$Lu = \sum_{i,j=1}^{n} a_{ij}\frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} b_i \frac{\partial u}{\partial x_i} + cu - \frac{\partial u}{\partial t} = 0$$

in $\Omega = R^n \times (0, \infty)$, where $x = (x_1, \cdots, x_n)$ is a point of the n-dimensional Euclidean space R^n, $t \in (0, \infty)$ the time-variable and $a_{ij} = a_{ji}, b_i$ and c are functions defined in Ω. In this paper, we have some interests in treating behavior of the continuous solution u of the Cauchy problem

$$(1) \qquad \begin{cases} Lu = 0 & \text{in } \Omega, \\ u(x, 0) = f(x) & \text{in } R^n. \end{cases}$$

In the case where $c \leqq 0$ in Ω, some results were obtained by many authors. For instance, we can prove the following.

Suppose that coefficients of the operator L satisfy the following condition in Ω:

$$(2) \qquad \begin{cases} 0 < \sum_{i,j=1}^{n} a_{ij}\xi_i\xi_j \leqq K_1(|x|^2+1)^{1-\lambda}|\xi|^2 \\ \qquad\qquad \text{for any real vector } \xi = (\xi_1, \cdots, \xi_n) \neq 0, \\ |b_i| \leqq K_2(|x|^2+1)^{1/2}, \quad (i = 1, \cdots, n), \\ c \leqq 0 \end{cases}$$

for some positive K_1, K_2 and $\lambda \in [0, \infty)$. Further, suppose that there exists a positive function $H(x)$ in R^n such that $LH \leqq -\delta$ in R^n for a positive constant δ and such that $H(x)$ tends to infinity as $|x|$ tends to infinity. If a continuous function $u = u(x, t)$ in $\overline{\Omega} = R^n \times [0, \infty)$, satisfying

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