Tôhoku Math. Journ.
23(1971), 289-299.

ON THE FUNCTIONAL EQUATION $\sum_{i=0}^{p} a_i f_i^{n_i} = 1$

NOBUSHIGE TODA*)

(Received Dec. 11, 1970)

1. Let $a_0, \cdots, a_p$ $(p \geqslant 1)$ be $p+1$ meromorphic functions in $|z| < R(\leqslant \infty)$ and $n_0, \cdots, n_p$ positive integers. In this paper we consider whether the functional equation

$$\sum_{i=0}^{p} a_i f_i^{n_i} = 1$$

has holomorphic solutions $f_0, \cdots, f_p$ in $|z| < R$.

Recently, Yang [10] has proved the following

THEOREM A. *The functional equation*

$$a(z) f^m(z) + b(z) g^n(z) = 1 \tag{1}$$

(a, b, f, g meromorphic in $|z| < \infty$, m and n integers $\geqslant 3$) cannot hold, if

$$T(r, a) = o(T(r, f)), \quad T(r, b) = o(T(r, g)), \tag{2}$$

unless $m = n = 3$.

If $f(z)$ and $g(z)$ are entire and (2) *holds, then* (1) *cannot hold, even if* $m = n = 3$.

This is a generalization of the case $a \equiv b \equiv 1$ treated by Montel [7], Jategaonkar [5] and Gross [2, 3]. Further, Iyer [4], Jategaonkar and Gross considered many other cases.

We will generalize the latter half of Theorem A and give some consequences of the generalizations.

It is assumed that the reader is familier with the fundamental concepts of Nevanlinna's theory of meromorphic functions and the symbols $m(r, f)$, $N(r, f)$, $\overline{N}(r, f)$, $T(r, f)$, etc. (see [11]).

*) Supported in part by the Sakkokai Foundation.