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THE HAUSDORFF DIMENSION OF THE SINGULAR SETS OF COMBINATION GROUPS

Dedicated to Professor Y. Tōki on his sixtieth birthday

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Introduction. It is well known that the Hausdorff dimension of the singular set of a Schottky group is positive and smaller than 2 ([5]). Moreover, it is also shown that there exists a Schottky group with a fundamental domain bounded by four circles whose singular set has positive 1-dimensional Hausdorff measure ([2]). Recently one of the present authors proved the existence of Kleinian groups whose singular sets have positive (3/2)-dimensional measure ([4]). The fundamental domains of groups mentioned above are domains bounded by a finite number of mutually disjoint circles. From these facts, it is natural for us to set up the following problem: Does the Hausdorff dimension of the singular sets of finitely generated Kleinian groups with fundamental domains bounded by mutually disjoint circles climb up, when the number of the boundary circles increases? What is the supremum of the Hausdorff dimensions determined by all such groups?

In this paper we shall give the result that the Hausdorff dimension increases strictly according to increment of the number of boundary circles.

1. Statement of theorem. Let us denote by B the unbounded domain in the complex plane whose boundary consists of $N(\geqq 1)$ mutually disjoint circles $\{K_i\}_{i=1}^{N}$. We shall form a discontinuous group of linear transformations with the fundamental domain B in the following. Take p pairs of boundary circles from $\{K_i\}_{i=1}^{N}$ and denote them by $\{H_i, H_i'\}_{i=1}^{p}$. Let S_i $(1 \leqq i \leqq p)$ be a hyperbolic or loxodromic transformation which transforms the outside of H_i onto the inside of H_i'. We denote by S_i^{-1} the inverse transformation of S_i. Consider the $N - 2p(\geqq 0)$ remaining boundary circles among $\{K_i\}_{i=1}^{N}$ and denote them by $\{K_j^*\}_{j=1}^{q}$, where $N = 2p + q$. Let S_j^* $(1 \leqq j \leqq q)$ be an elliptic transformation with period 2 which transforms the outside of K_j^* onto the inside of K_j^*. A group G, generated by $\{S_i\}_{i=1}^{p}$ and $\{S_j^*\}_{j=1}^{q}$, is a discontinuous group with a fundamental domain B. In the special case of $N = 2p$, G is a Schottky group, which contains the elementary group of the case $p = 1$. If N is odd, there exists