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AUTOMORPHISM GROUPS OF HOPF SURFACES

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Introduction. Let $GL(2, \boldsymbol{C})$ be the group of non-singular (2×2)-matrices. An element $u = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ of $GL(2, \boldsymbol{C})$ operates on $\boldsymbol{C}^2$ as follows:

$$(z, w) \to (az + bw, cz + dw) .$$

Let M be a subset of $GL(2, \boldsymbol{C})$ defined by

$$M = \left\{ \begin{pmatrix} \alpha & t \\ 0 & \beta \end{pmatrix} \middle| \alpha, \beta, t \in \boldsymbol{C}, 0 < |\alpha| < 1, 0 < |\beta| < 1 \right\} .$$

Then M is a complex manifold. Let 0 be the origin of $\boldsymbol{C}^2$. We put $W = \boldsymbol{C}^2 - 0$. Let $u \in M$. Then u defines a properly discontinuous group

$$G_u = \{u^n \mid n \in \boldsymbol{Z}\}$$

of automorphisms (holomorphic isomorphisms) without fixed point of W. Hence we have a complex manifold

$$V_u = W/G_u .$$

V_u is easily seen to be compact. It is called a Hopf surface. It can be shown that the collection

$$\{V_u\}_{u \in M}$$

forms a complex analytic family (X, π, M). We denote by $\mathrm{Aut}(V_u)$ the group of automorphisms of V_u.

The purpose of this note is prove the following theorem.

THEOREM. *The disjoint union*

$$A = \coprod_{u \in M} \mathrm{Aut}(V_u)$$

admits a (reduced) analytic space structure such that

1) $\lambda: A \to M$ *is a surjective holomorphic map, where* λ *is the canonical projection,*

2) *the map*

$$A \times_M X \to X$$

defined by