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PLURIHARMONIC BOUNDARY VALUES

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Abstract. Let $\Omega = \{\rho < 0\}$ be a domain with C^3-boundary $\Gamma = \{\rho = 0\}$. For a large class of domains, the functions $u \in C^3(\Gamma)$ which are the restrictions of pluriharmonic functions on Ω are characterized as the solutions of a system of partial differential equations.

I. Introduction. Let $\Omega = \{\rho < 0\}$ be a bounded domain in $\boldsymbol{C}^n (n \geqq 2)$ with connected C^3-boundary $\Gamma = \{\rho = 0\}$, grad $\rho \neq 0$ on Γ. A function $f \in C^1(\Gamma)$ can be extended to an analytic function F on Ω if and only if it satisfies the tangential Cauchy-Riemann equations:

$$(1) \qquad \bar{\partial}\rho \wedge \bar{\partial} f = 0$$

on Γ (see [1], [3]). We will give an analogous system for pluriharmonic functions. It will also be pointed out that the Neumann conditions for the $\partial\bar{\partial}$-operator give a simple characterization of pluriharmonic functions although these conditions involve derivatives normal to Γ.

Let $d = 1/2(\bar{\partial} + \partial)$ and $d^c = 1/2i(\bar{\partial} - \partial)$ denote the real and imaginary parts of $\bar{\partial}$. It will be shown here that for certain domains Ω, a function $u \in C^3(\Gamma)$ can be extended to a pluriharmonic function U on Ω if and only if there exists a function $\alpha \in C^1(\Gamma)$ such that:

$$(2) \qquad d\rho \wedge d^c\rho \wedge dd^c u = \alpha d\rho \wedge d^c\rho \wedge dd^c\rho$$

$$(3) \qquad d\rho \wedge dd^c u = d\rho \wedge d\alpha \wedge d^c\rho + \alpha d\rho \wedge dd^c\rho \,.$$

Since the expression $dd^c u$ does not depend only on the values of u on Γ, the equations (2) and (3) are to be interpreted in the following sense. A C^3 extension u_1 of u to a neighborhood of Γ is picked, and the same extension u_1 is substituted into both equations (2) and (3).

If u extends to a pluriharmonic function U on Ω, then (2) and (3) are satisfied. For any extension u_1 will have the form $u_1 = U + a\rho$, and equation (2) will yield $a = \alpha$ since $dd^c U = 0$. With $a = \alpha$, equation (3)

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