Tôhoku Math. Journ.
27 (1975), 601-616.

NONHOMOGENEOUS ELLIPTIC SYSTEMS AND SCATTERING

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(Received December 5, 1974)

Abstract. We prove the main conclusions of scattering theory for a class of first order elliptic systems in which the free (unperturbed) system is not homogeneous. The perturbed system need not be essentially self adjoint, and the assumptions on the perturbation are mild. The Dirac operator is a special case of the systems considered.

1. Introduction. We examine scattering theory for elliptic systems of the form

$$H(x, D) = E(x)^{-1}(A_0(D) - B(x)) , \tag{1.1}$$

where

$$A_0(D) = \sum_{j=1}^{m} A_0^j D_j + B_0 , \tag{1.2}$$

$x = (x_1, \cdots, x_n)$, $E(x)$, $B(x)$, B_0 and the A_0^j are hermitian $m \times m$ matrices and $D_j = \partial/i\partial x_j$. This is compared with the free system

$$H_0(D) = E_0^{-1} A_0(D) , \tag{1.3}$$

where E_0 is also hermitian. The aim of the paper is to obtain the main conclusions of scattering theory under minimal conditions on the perturbation $B(x)$. Our assumptions on this matrix will not be sufficient to make $H(x, D)$ essentially self adjoint. This creates several technical difficulties. The situation is further complicated by the appearance of the matrix B_0 in the free system, which spoils homogeniety. The fact that $E(x) \risingdotseq E_0$ adds to the difficulty.

To obtain a self adjoint extension of $H(x, D)$ we need a criterion involving two Hilbert spaces. For this purpose we generalized a theorem due to Kato [1] for one Hilbert space. We obtain this theorem without restricting the size of the perturbation or requiring it to be compact (see Section 2). We use the factored perturbation technique for two Hilbert spaces developed in [12]. In order to get maximum benefit from this technique we introduced pseudo-differential operators of order 1/2 to make the factorization as even as possible. This introduces other technical difficulties which require special attention (see Section 3).