Tôhoku Math. Journ.
27 (1975), 569-599.

# CONSTRUCTING MANIFOLDS BY HOMOTOPY EQUIVALENCES II

Browder-Novikov-Wall Type Obstruction to Constructing *PL*- and Topological Manifolds from Homology Manifolds

HAJIME SATO

(Received June 17, 1974)

**0. Introduction.** Let $M$ be a homology manifold of dimension $n \geqq 5$. If $\partial M \neq \emptyset$, suppose that a neighborhood of $\partial M$ is a *PL*-manifold. In the previous paper [8], we have defined the obstruction $\lambda(M) = \sum_{\sigma:(n-4)\text{-simplexes}} \sigma \otimes \{Lk(\sigma)\}$ in

$$H_{n-4}(M; \mathscr{H}^3) ,$$

where $\mathscr{H}^3$ is the group of 3-dimensional *PL*-homology spheres modulo those which are the boundary of an acyclic *PL*-manifold. If the obstruction vanishes, then $M$ is pseudo cellular equivalent and simple homotopy equivalent to a *PL*-manifold with the same boundary. In this paper, we search for a *PL*-manifold or a topological manifold which is simple homotopy equivalent or $(\pi_1, H_*)$-equivalent to $M$. We call a map a $(\pi_1, H_*)$-equivalence if it induces isomorphisms of the fundamental groups and the homology groups of all dimensions.

We have a surjective homomorphism

$$i: \mathscr{H}^3 \rightarrow Z_2 .$$

Let $\beta: H_{n-4}(M; Z_2) \rightarrow H_{n-5}(M; Z)$ be the integral Bockstein homomorphism. Then we have the composition

$$\beta \circ i_*: H_{n-4}(M; \mathscr{H}^3) \rightarrow H_{n-5}(M; Z) .$$

This composition was firstly considered by Sullivan [20].

Our first theorem is as follows.

THEOREM 1. *If the obstruction*

$$\beta \circ i_*(\lambda(M)) \in H_{n-5}(M; Z)$$

*is zero, and if a surgery obstruction in the Wall group*

$$L_n(\pi_1(M), \omega)$$

*is zero, $M$ is relatively simple homotopy equivalent to a PL-manifold*

Supported partially by the Sakkokai Foundation.