Tôhoku Math. Journ.
28 (1976), 135-143.

ADJOINT EQUATIONS OF AUTONOMOUS LINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE RETARDATIONS

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(Received August 25, 1975)

1. Introduction. Let $\rho \geqq r \geqq 0$, $p \geqq 1$ be given real numbers (ρ may be $+\infty$) and $g(\theta)$ be Lebesgue integrable, positive and nondecreasing on $[-\rho, 0]$, where $[-\rho, 0]$ denotes $(-\infty, 0]$ when $\rho = +\infty$. Let $\mathscr{B} = \mathscr{B}([-\rho, 0], \boldsymbol{C}^d)$ be the Banach space of functions ϕ mapping $[-\rho, 0]$ into $\boldsymbol{C}^d$, the complex d-dimensional column vector space, which are Lebesgue measurable on $[-\rho, 0]$, are continuous on $[-r, 0]$ and have the property such that

$$\|\phi\| = \left[\sup_{-r\leqq\theta\leqq 0} |\phi(\theta)|^p + \int_{-\rho}^{0} |\phi(\theta)|^p g(\theta)d\theta\right]^{1/p} < \infty ,$$

where $|v|$ denotes a norm of v in $\boldsymbol{C}^d$. We shall discuss the adjoint equation of a linear functional differential equation

$$\frac{dx}{dt} = f(x_t) , \tag{1.1}$$

where f is a bounded linear operator on $\mathscr{B}$ into $\boldsymbol{C}^d$. Denote by ${}^T v$ the transposed vector of $v \in \boldsymbol{C}^d$ and by ${}^T\boldsymbol{C}^d$ the space $\{{}^T v; v \in \boldsymbol{C}^d\}$. For a given function ϕ mapping $[-\rho, 0]$ into $\boldsymbol{C}^d$, the function ϕ^* mapping $[0, \rho]$ into ${}^T\boldsymbol{C}^d$ is defined by $\phi^*(s) = {}^T\phi(-s)$, $s \in [0, \rho]$. For a family $\mathscr{F}$ of those functions ϕ, set $\mathscr{F}^* = \{\phi^*; \phi \in \mathscr{F}\}$. For a function x defined on $[t-\rho, t]$ (or $[t, t+\rho]$), designate by x_t (or x^t) the function on $[-\rho, 0]$ (or $[0, \rho]$) such that $x_t(\theta) = x(t+\theta)$, $\theta \in [-\rho, 0]$ (or $x^t(s) = x(t+s)$, $s \in [0, \rho]$).

Now consider a linear functional differential equation for a row vector y

$$\frac{dy}{dt} = -(y^t)\overline{f|} . \tag{1.2}$$

The symbol $\overline{f|}$ denotes the operator on $\mathscr{B}^*$ naturally induced by f which operates on $\mathscr{B}^*$ to the right (see (3.6) and (3.7)). However, we restrict the domain of $\overline{f|}$ on a space $\mathscr{X}^*$ such that $\mathscr{X}$ can be imbedded continuously in $\mathscr{B}$ and that for any $\xi \in \mathscr{X}^*$ and any $\phi \in \mathscr{B}$, the convolution