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HYPONORMAL OPERATORS IN VON NEUMANN ALGEBRAS

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1. In [2], Foias and Kovàcs proved that a finite von Neumann algebra is characterized by the terms of contractions in it. The essential part of their proof is a trace argument. Our interest is how to prove this without using such trace argument because we can not use it in more general algebra (for example, AW^*-algebra).

We say that a bounded linear operator T on a Hilbert space H is hyponormal if $T^*T \geqq TT^*$, or equivalently, if $\|Tx\| \geqq \|T^*x\|$ for all $x \in H$. If T is an invertible hyponormal operator on H, then T^{-1} is also hyponormal and $T^{*-1}T$ is a contraction (i.e., $\|T^{*-1}T\| \leqq 1$) in $R(T)$ which denotes the von Neumann algebra generated by T. On the other hand, for a contraction T on H, we can construct a hyponormal operator in $R(T)$ associated with T. And there is an interesting and useful relation between hyponormal operators and contractions in a von Neumann algebra. From this point of view, it is important to study the behaviour of hyponormal operators in a von Neumann algebra and, in §2, we shall study such relation between hyponormal operators and contractions and show some applications of it.

The most effective technique being used in this section is the decomposition theorem (called canonical decomposition) of a contraction T and moreover such decomposition can be done in $R(T)$. This follows from the characterization (due to [6] and also [4]) of the subspace $H^{(u)}$ on which the unitary part of T acts as follows;

$$\begin{aligned} H^{(u)} &= \{x \in H : \|T^k x\| = \|x\| = \|T^{*k}x\|, k = 1, 2, \cdots\} \\ &= \bigcap_{k=1}^{\infty} \{x \in H : T^{*k}T^k x = x = T^k T^{*k} x\} . \end{aligned}$$

Concerning this decomposition, it is known that every bounded linear operator T on H can be written uniquely as the direct sum of a normal part and a completely non-normal part of T. But, in this case, known characterizations of $H^{(n)}$ on which the normal part of T acts is not so precise as that of $H^{(u)}$ for the case T is a contraction. In §3, we shall show that the subspace $H^{(n)}$ for the case where T is hyponormal is characterized as follows;