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ON EXTENDIBILITY OF ISOMORPHISMS OF CARTAN CONNECTIONS AND BIHOLOMORPHIC MAPPINGS OF BOUNDED DOMAINS

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Introduction. In 1972 the author proved in his preprint [5] that any biholomorphic mapping of two smooth, strongly pseudo-convex bounded domains is smooth up to the boundary, under somewhat too strong assumption on the boundary behavior of the Bergman kernel. Fortunately Fefferman proved a relevant smoothness theorem in [2]. He also proved the extendibility of biholomorphic mappings mentioned above by analysing directly the behavior of geodesics (of the Bergman geometry) near the boundary. The author's method is first to reduce the problem to the heigher dimensional case in which the biholomorphic map is extended up to almost all part of the boundary, and then to extend it to the remaining part applying an extendibility theorem for Cartan connections. Since this method clarifies the geometric essence from a different point of view and since this last theorem is interesting in its own right, it seems to be worth reproducing here the argument of [5] shortly.

1. Extendibility of Isomorphisms of Cartan Connections. We begin with some elementary study of Riemannian geometry. Throughout this section we assume the differentiability of class C^k, $k \geqq 1$, so that "smooth", "diffeomorphism" mean "C^k smooth", "C^k diffeomorphism". Let M be a Riemannian manifold and C a closed submanifold of M of codimension not less than 2. Given two points $x, y \in M \backslash C$, we can define two distances between x and y, the one with respect to M and the other with respect to $M \backslash C$ which is regarded as a new Riemannian manifold. By the codimension reason, curves in M having end points fixed in $M \backslash C$ can be approximated smoothly by those lying in $M \backslash C$. Thus the two distances above coincide. In particular

LEMMA 1.1. *The metric completions of $M \backslash C$, M coincide.*

As is well known, a one to one correspondence of two Riemannian manifolds is an isometry (metric-tensor-preserving diffeomorphism) if and only if it is distance-preserving. (See e.g. Helgason [3], Theorem 11.1)