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ALMOST CONTACT STRUCTURES ON BRIESKORN MANIFOLDS

SHIGEO SASAKI AND TOSHIO TAKAHASHI

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1. Introduction. Let $a_0, a_1, \cdots, a_n$ be positive integers and let $X^{2n} = X^{2n}(a_0, a_1, \cdots, a_n)$ be the algebraic variety given by $X^{2n}(a_0, a_1, \cdots, a_n) = \{z = (z_0, z_1, \cdots, z_n) \in C^{n+1} | z_0^{a_0} + z_1^{a_1} + \cdots + z_n^{a_n} = 0\}$. The only possible singularity of X^{2n} is the origin 0 of C^{n+1}, and $B^{2n} = B^{2n}(a_0, a_1, \cdots, a_n) = X^{2n}(a_0, a_1, \cdots, a_n) - \{0\}$ is a complex hypersurface of C^{n+1}. Let $\Sigma^{2n-1} = \Sigma^{2n-1}(a_0, a_1, \cdots, a_n)$ be the intersection of B^{2n} and the unit sphere $S^{2n+1} = \{z \in C^{n+1} | z_0\bar{z}_0 + z_1\bar{z}_1 + \cdots + z_n\bar{z}_n = 1\}$, which we call a Brieskorn manifold.

A Brieskorn manifold Σ^{2n-1} is a real hypersurface of the complex manifold B^{2n}, and it is also a submanifold of the unit sphere S^{2n+1} with codimension 2. K. Abe [1] introduced an almost contact structure on Σ^{2n-1} by using a property that $\Sigma^{2n-1} \times R$ is diffeomorphic with B^{2n}, and discussed about the non-regularity of the almost contact structure. On the other hand, C. J. Hsu and one of the authors [6] introduced a contact structure on Σ^{2n-1} by using a property that Σ^{2n-1} is a submanifold of B^{2n} and S^{2n+1}, and gave a necessary and sufficient condition that the almost contact structure given by K. Abe and the almost contact structure given in [6] coincide. K. Abe and J. Erbacher [2] also introduced contact structures on a wide class of submanifolds which contain Brieskorn manifolds as a special case.

In this note, first we give a simplified definition of the almost contact structure introduced by K. Abe, and the criterions for its non-regularity and regularity. Secondly we show that our almost contact structure is normal.

2. Definition of an almost contact structure for Brieskorn manifold Σ^{2n-1}. Let $\{f_s\}(s \in R)$ be the 1-parameter group of holomorphic transformations of the complex manifold B^{2n} given by

$$f_s(z_0, z_1, \cdots, z_n) = (e^{b_0 s}z_0, e^{b_1 s}z_1, \cdots, e^{b_n s}z_n) , \tag{2.1}$$

where $b_0 = m/a_0, b_1 = m/a_1, \cdots, b_n = m/a_n$ and m is the L.C.M. of $a_0, a_1, \cdots, a_n$. Let α be the vector field on B^{2n} which is induced by the 1-parameter group $\{f_s\}(s \in R)$. It is easy to see that α is transversal and $J\alpha$ is tangent to the Brieskorn manifold Σ^{2n-1}, where J is the induced complex structure