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UNIQUENESS OF THE NORMAL CONNECTIONS AND CONGRUENCE OF ISOMETRIC IMMERSIONS

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Let f be an isometric immersion of a Riemannian manifold M into a Riemannian manifold $\tilde{M}$ of constant curvature and let N_f be the normal bundle. The normal connection is a metric linear connection in the bundle N_f which satisfies the Codazzi equation for the second fundamental form α. The first aim of the present paper is to prove the following result: in the case where the first normal space $N_1(x)$ coincides with the normal space $N(x)$ at each point x of M, a metric linear connection in the bundle N_f which satisfies the equation of Codazzi type coincides with the normal connection. This fact can be derived as a special case of the general treatment of connections in the bundles of normal spaces of higher orders as given by O. Kowalski [6], but simplicity of the result under our assumption is remarkable. Indeed, we shall define the torsion tensor of an arbitrary linear connection in the bundle N_f in such a way that the normal connection can be characterized, still under the assumption that $N_1(x) = N(x)$ for every point x, as a unique metric linear connection in N_f whose torsion tensor is 0. This then is an analogue of the uniqueness theorem of the Riemannian connection as a linear metric connection with zero torsion in the tangent bundle of a Riemannian manifold.

The second aim of the paper is to apply the result above to obtain congruence theorems for isometric immersions which satisfies $N_1(x) = N(x)$ for all points x or whose second fundamental forms are parallel. Isometric immersions into a Euclidean space with parallel second fundamental forms have been essentially determined by D. Ferus [2], [3], [4], and our result has a close bearing on part of the proof of the main result in [4].

1. Uniqueness of the normal connection. Let f be an isometric immersion of a Riemannian manifold M into a Riemannian manifold M of constant curvature. For each point x of M, let $N(x)$ be the normal space and let N_f be the normal bundle. The second fundamental form

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