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COMMUTATIVE NORMAL *-DERIVATIONS III*),**)

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1. Introduction. In the present paper, we shall continue the study of commutative derivations. We shall show general methods to obtain all *KMS*-states. Also we shall generalize the theorem in [6] to commutative derivations with infinite range interaction, and we shall examine the relation between this theorem and Dyson conjecture [1] and Kac-Thompson conjecture [1] concerning one-dimensional Ising ferromagnet.

2. Theorems. Let $\mathfrak{A}$ be a uniformly hyperfinite C^*-algebra, and let δ be a normal *-derivation in $\mathfrak{A}$-i.e., there is an increasing sequence of finite type I subfactors $\{\mathfrak{A}_n\}$ in $\mathfrak{A}$ such that $\bigcup_{n=1}^{\infty}\mathfrak{A}_n$ is dense in $\mathfrak{A}$ and the domain $\mathfrak{D}(\delta)$ of δ is $\bigcup_{n=1}^{\infty}\mathfrak{A}_n$. Then there is a sequence of self-adjoint elements $\{h_n\}$ in $\mathfrak{A}$ such that $\delta(a) = i[h_n, a]$ $(a \in \mathfrak{A}_n)$ $(n = 1, 2, \cdots)$. Suppose that δ is commutative -i.e., we can choose (h_n) as a commutative family.

Let $\mathfrak{L}_n$ be the C^*-subalgebra of $\mathfrak{A}$ generated by $\mathfrak{A}_n$ and h_n. Suppose that $h_n \in \bigcup_{m=1}^{\infty}\mathfrak{A}_m (n = 1, 2, \cdots)$; then $\mathfrak{L}_n$ is finitedimensional. Let $(p_{n,j})_{j=1}^{m(n)}$ be the family of all minimal projections in the center Z_n of $\mathfrak{L}_n$. Then $\mathfrak{L}_n = \sum_{j=1}^{m(n)} \mathfrak{L}_n p_{n,j}$. Let $\{\rho(t)\}$ be the strongly continuous one-parameter subgroup of *-automorphisms on $\mathfrak{A}$ corresponding to δ (cf. [5]). Then $\rho(t)(a) = e^{+tih_n}\, a e^{-tih_n}$ $(a \in \mathfrak{L}_n)$ and so $\rho(t)\mathfrak{L}_n p_{n,j} = \mathfrak{L}_n p_{n,j}$.

Now we shall show the following theorem.

THEOREM 1. *Suppose* $h_n \in \bigcup_{m=1}^{\infty}\mathfrak{A}_m (n = 1, 2, \cdots)$ *and let* $\psi_{n,j,\beta}(x) = \tau(xe^{-\beta h_n}p_{n,j})/\tau(e^{-\beta h_n}p_{n,j})$ $(x \in \mathfrak{A})$, *where* $-\infty < \beta < +\infty$ *and* τ *is the unique tracial state on* $\mathfrak{A}$. *Let*

$$G_\beta = \{\psi_{n,j,\beta} \mid 1 \leqq j \leqq m(n) \text{ and } n = 1, 2, 3, \cdots\}.$$

Then the set of all accumulation points in the $\sigma(\mathfrak{A}^*, \mathfrak{A})$-*closure* $\bar{G}_\beta$ *of* G_β *in the state space of* $\mathfrak{A}$ *contains all extreme KMS states on* $\mathfrak{A}$ *for* $\{\rho(t)\}$ *at* β. *Moreover every accumulation point of* $\bar{G}_\beta$ *in the* $\sigma(\mathfrak{A}^*, \mathfrak{A})$-*topology is a KMS state for* $\{\rho(t)\}$ *at* β, *where* $\mathfrak{A}^*$ *is the dual Banach space of* $\mathfrak{A}$.

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