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ON THE ABSOLUTE NÖRLUND SUMMABILITY FACTORS OF THE CONJUGATE SERIES OF A FOURIER SERIES

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1. Let $\{s_n\}$ denote the n-th partial sum of a given infinite series $\sum a_n$. Let $\{p_n\}$ be a sequence of constants, real or complex, and let

$$P_n = p_0 + p_1 + \cdots + p_n\ ; \quad P_{-k} = p_{-k} = 0\ , \quad \text{for} \quad k \geqq 1\ .$$

The sequence $\{t_n\}$, given by

$$t_n = \frac{1}{P_n}\sum_{k=0}^{n} p_{n-k}s_k = \frac{1}{P_n}\sum_{k=0}^{n} P_{n-k}a_k\ , \quad (P_n \neq 0)\ , \tag{1.1}$$

defines the Nörlund means of the sequence $\{s_n\}$ generated by the sequence $\{p_n\}$.

Then, the series $\sum a_n$ is said to be summable $|N, p_n|$, if the sequence $\{t_n\}$ is of bounded variation, that is, the series

$$\sum_n |t_n - t_{n-1}| \tag{1.2}$$

is convergent.

In the special cases in which $p_n = \Gamma(n+\alpha)/\Gamma(\alpha)\Gamma(n+1)$, $\alpha > 0$, and $p_n = 1/(n+1)$, summability $|N, p_n|$ are the same as the summability $|C, \alpha|$ and the absolute harmonic summability, respectively.

Let $f(t)$ be a periodic function with period 2π and integrable (L) over $(-\pi, \pi)$. We assume without any loss of generality that the Fourier series of $f(t)$ is given by

$$\sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) = \sum_{n=1}^{\infty} A_n(t) \tag{1.3}$$

and $\int_{-\pi}^{\pi} f(t)dt = 0$.

The series "conjugate" to (1.3) is

$$\sum_{n=1}^{\infty} (b_n \cos nt - a_n \sin nt) = \sum_{n=1}^{\infty} B_n(t)\ .$$

We write