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A DISTORTION THEOREM IN K-QUASICONFORMAL MAPPINGS OF THE n-BALL

KAZUO IKOMA

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As is well known, the plane Teichmüller's modulus theorem which estimates for the modulus of the plane ring separating a pair of points 0 and γ from δ and ∞ played an important role in dealing with distortion problems in function theory. Lehto-Virtanen [4] and A. Mori [5] modified this theorem so as to be suitable for further applications to distortion problems, that is to say, they estimated for the modulus of the plane ring separating a pair of points α and β from 0 and ∞. The main point of the proof for this modification is that a ring separating α and β from 0 and ∞ is doubled by means of two branches of the inverse of the mapping $z = w^2$ and then a ring whose boundary components are separated by doubled rings is transformed into another ring to which the plane Teichmüller's modulus theorem can be applied.

Now, in order to estimate for the modulus of a ring in n-space ($n \geqq 3$) separating a pair of points α and β from 0 and ∞, the ring is to be transformed into another ring so that one may be able to use Teichmüller's modulus theorem in n-space which estimates for the modulus of a ring separating a pair of points 0 and γ from δ and ∞. To this purpose, an appropriate auxiliary K-quasiconformal mapping is required, where K can not be equal to 1. The reason can be seen from the above mentioned situation in the 2-dimensional case and the familiar result*) that every 1-quasiconformal mapping of a domain in space is nothing but a restriction of a Möbius transformation to the domain. In this paper, we estimate in Theorem 1, for the modulus of a ring in n-space separating a pair of points α and β from 0 and ∞, and as its application, show in Theorem 2, a distortion result of the Hölder type for K-quasiconformal mappings of the unit ball in n-space.

1. The modulus of a ring and K-quasiconformality. A domain R in the Möbius n-space**) is called a ring if its complement has exactly

*) See, for instance, Rešetnjak [7].

**) The Möbius n-space means the one point compactification of the euclidean n-space obtained by adding the point at infinity.