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THE HARDY SPACES ASSOCIATED WITH A PERIODIC FLOW ON A VON NEUMANN ALGEBRA

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0. Introduction. In the study of non-self adjoint subalgebras of von Neumann algebras, several attempts have been made to generalize a theory of function algebras to non-commutative cases. For instance, a theory of subdiagonal algebras was presented by Arveson as an analogue of weak*-Dirichlet algebras in [1]. In this paper we present a method to construct the Hardy spaces associated with a periodic flow on a von Neumann algebra. The method is based on the theory of spectral subspaces for a flow which has been investigated by many authors [2, 3, 9]. Kawamura and Tomiyama [5] studied the Hardy spaces associated with a flow and discussed related situations in operator algebras.

Let T be the unit circle. We define a flow β with period 2π of $L^\infty(T)$ as follows: $\beta_t f(z) = f(e^{-it}z)$, $t \in R$, $z \in T$, $f \in L^\infty(T)$. Let M be a von Neumann algebra acting on a Hilbert spaces H, M_* its predual and α a periodic flow with period 2π on M. Then M, M_*, H and α correspond to $L^\infty(T)$, $L^1(T)$, $L^2(T)$ and β, respectively. Then, in view of the role played by the Hardy spaces H^p in $L^p(T)$, we construct $H^p(\alpha)(p = 1, 2, \infty)$. In particular $H^\infty(\alpha)$ is not only a σ-weakly closed non-self adjoint subalgebra but also turns to be a maximal subdiagonal algebra. If there exists an ergodic, periodic flow on M, then M is generated by a single unitary operator. In this case we use the Cesaro mean defined by a periodic flow on M. If M is σ-finite, we have a decomposition of a von Neumann algebra with respect to a periodic flow and reconsider a part of Takesaki's consequence in [10] for a von Neumann algebra with a homogeneous periodic state.

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1. Preliminaries. Let M be a von Neumann algebra acting on a Hilbert space H, M_* its predual and $\alpha_t (t \in R)$ a flow on M, that is, a one-parameter group of *-automorphisms of M which is weak*-continuous in