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A NOTE ON MAUS' THEOREM ON RAMIFICATION GROUPS

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Introduction. Let k be a complete field under a discrete valuation with a perfect residue field $\bar{k}$ of characteristic $p \neq 0$, and let K/k be a fully ramified finite Galois extension with Galois group G. Let G_i denote the i-th ramification group of G. Then it is well known that the sequence $G = G_0 \supseteqq G_1 \supseteqq \cdots \supseteqq G_i \supseteqq G_{i+1} \supseteqq \cdots$ has the following properties:

G_i is normal in G for $i \geqq 0$, and there exists $i_0 > 0$ such that $G_i = 1$ for $i \geqq i_0$; G_0/G_1 is a cyclic group of order prime to p; for $i \geqq 1$, G_i/G_{i+1} is an elementary abelian p-group contained in the center of G_1/G_{i+1}; as a G_0/G_1-module, G_i/G_{i+1} is the direct sum of irreducible submodules which are isomorphic each other, for $i \geqq 1$.

Maus [3] has proved the 'inverse' of the above when k is a finite algebraic extension of the field of p-adic numbers $\boldsymbol{Q}_p$ and when k is of characteristic p, by using local class field theory and Artin-Schreier theory, respectively.

The purpose of this paper is to show that Maus' theorem is also valid when k is a complete field of characteristic 0 under a discrete valuation with a perfect residue field $\bar{k}$ of characteristic p, using Kummer theory.

For a Galois extension K of k, the *sequence of ramification groups* of K/k means the descending sequence of all ramification groups of K/k, without taking ramification numbers into account.

MAUS' THEOREM. *Let k be a complete field of characteristic* 0 *under a discrete valuation with a perfect residue field $\bar{k}$ of characteristic p and with absolute ramification order e_k, i.e., $e_k = \mathrm{ord}_k(p)$, where ord_k is the normalized additive valuation of k. Let $G = G^{(0)} \supseteqq G^{(1)} \supsetneqq \cdots \supsetneqq G^{(r)} \supsetneqq G^{(r+1)} = 1$ be the sequence of finite groups satisfying the following*:

(i) *$G^{(i)}$ is a normal subgroup of G for $i = 0, 1, \cdots, r$;*

(ii) *$G^{(0)}/G^{(1)}$ is a cyclic group of order prime to p;*

(iii) *$G^{(i)}/G^{(i+1)}$ is an elementary abelian p-group contained in the center of $G^{(1)}/G^{(i+1)}$ for $i \geqq 1$;*

(iv) *As a $G^{(0)}/G^{(1)}$-module, $G^{(i)}/G^{(i+1)}$ is the direct sum of irreducible submodules which are isomorphic each other, for $i = 1, 2, \cdots, r$. Then*