Tôhoku Math. Journ.
29 (1977), 1-8.

ON THE ISOMETRIC STRUCTURE OF RIEMANNIAN MANIFOLDS OF NON-NEGATIVE RICCI CURVATURE CONTAINING A COMPACT HYPERSURFACE WITHOUT FOCAL POINT

KIYOTAKA II

(Received July 2, 1974)

1. Introduction. In their paper [2], J. Cheeger and D. Gromoll proved the following:

THEOREM (Cheeger-Gromoll). *Let M be a connected, complete and non-compact Riemannian manifold of non-negative Ricci curvature. If M contains a line, then M is isometric to the Riemannian product $N\times \boldsymbol{R}$, where N is a totally geodesic hypersurface in M.*

Recall that a line is a normal geodesic $l:(-\infty, \infty)\rightarrow M$, any segment of which is minimal.

The above theorem says that the existence of suitable geometric objects in M determines the isometric structure of M. In the present paper, we shall consider the case where M contains a compact hypersurface without focal point. Our results are the following:

THEOREM A. *Let M be a connected, complete and non-compact Riemannian manifold of non-negative Ricci curvature. If M contains a compact hypersurface N without focal point, then N is totally geodesic, and M is isometric to a flat line bundle on N or on $N/\boldsymbol{Z}_2$.*

THEOREM B. *Let M be a connected, compact Riemannian manifold of non-negative Ricci curvature. If M contains a compact hypersurface N without focal point, then N is totally geodesic, and M is isometric to a Riemannian manifold $\perp_{[0,r]}N/i$.*

The Riemannian manifold $\perp_{[0,r]}N/i$ is defined as follows: For $r>0$, let $\perp_{[0,r]}N$ be a flat line bundle on N with fibre $[-r, r]$. Let i: $\perp_r N\rightarrow \perp_r N$ be a fixed-point free isometric involution on the boundary $\perp_r N$ of $\perp_{[0,r]}N$. Then identifying the boundary points u and $i(u)$, we obtain the Riemannian manifold $\perp_{[0,r]}N/i$.

2. Preliminaries. Let M be an n-dimensional connected and complete Riemannian manifold with Riemannian metric $\langle\ ,\ \rangle$ and Levi-Civita