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ON A GENERALIZATION OF THE HOPF FIBRATION, II*

(Complex structures on the products of generalized Brieskorn manifolds)

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This paper is the continuation of our previous paper under the same title [2], which we refer to as Part I. In Part I, we studied contact structures on the generalized Brieskorn manifolds as a generalization of the connection form of the Hopf fibration. Our objective in this paper is to study complex structures on the products of generalized Brieskorn manifolds. These complex structures may be considered as a generalization of the Calabi-Eckmann complex structures [6]. Therefore, we are particularly interested in studying those aspects of the complex manifolds which Calabi and Eckmann studied on the products of odd dimensional spheres. In fact, we show that our complex manifolds possess properties analogous to the Calabi-Eckmann manifolds.

Let Σ_1 and Σ_2 be generalized Brieskorn manifolds; see Part I for the definition and examples. First we show that $\Sigma_1 \times \Sigma_2$ admits a complex structure which is intimately related to the normal contact structures on Σ_1 and Σ_2. It is then shown that this complex structure admits no Kählerian structure and that $\Sigma_1 \times \Sigma_2$ is the total space of a holomorphic fibration on a complex analytic space $B_1 \times B_2$. This fibration, unlike the Calabi-Eckmann case, is not necessarily a fiber bundle; however, the fibers are elliptic curves. Our first main result is that any analytic subvariety of $\Sigma_1 \times \Sigma_2$ has the induced fibration over a complex analytic space. In particular, the fibers are the only irreducible analytic subvarieties of dimension 1; hence, they have no singular point. Earlier, Calabi and Eckmann have shown that their complex manifolds possess the same property [6]. However, their proof does not apply to our case directly, for $\Sigma_1 \times \Sigma_2$ may have non-vanishing middle homology and $B_1 \times B_2$ may not admit a projective imbedding.

Next, we show that many of the $\Sigma_1 \times \Sigma_2$'s admit infinitely many seemingly different complex structures. Indeed, many such examples are constructed. After introducing a criterion to distinguish these complex

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