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KNESER'S PROPERTY AND BOUNDARY VALUE PROBLEMS FOR SOME RETARDED FUNCTIONAL DIFFERENTIAL EQUATIONS

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1. Introduction. Recently, we [4] have extended an existence theorem of Nagumo for boundary value problems in second order ordinary differential equations (cf. [3], [4]). This paper is a further extension of our result to functional differential equations, and the proof given in this paper is simpler than that in [4].

As the phase space for retarded functional differential equations, Hale [1] first considered a Banach space of functions which satisfies some axioms. Recently, Hale and Kato [2] have improved the axioms for the phase space. We shall discuss the theory of functional differential equations in a semi-normed linear space as a phase space, and we shall assume some axioms which are essentially equivalent to those in [2]. Under these axioms, our results contain not only the theory for infinite delay but also the theory for finite delay and ordinary differential equations.

First we shall introduce the axioms for the phase space in Section 2, but our notations are somewhat different from those in [2], and we shall prove Kneser's property in Section 3 and apply this to a boundary value problem for some functional differential equation in Section 4. For a contingent functional differential equation, where the phase space is the class of all bounded and continuous functions, Kikuchi [5] proved the Kneser's property on R^n.

2. Preliminaries. Let B be a linear real vector space of functions mapping $(-\infty, 0]$ into R^n with the semi-norm $|\cdot|$. For any elements φ and ψ in B, $\varphi = \psi$ means $\varphi(\theta) = \psi(\theta)$ for all $\theta \in (-\infty, 0]$. The quotient space of B by the semi-norm $|\cdot|$, which is denoted by $\mathscr{B} = B/|\cdot|$, is a normed linear space with the norm $|\cdot|$ which is induced naturally by the semi-norm and for which we shall use the same notation. We do not assume $\mathscr{B}$ is a Banach space. The topology for B is naturally defined by the semi-norm, that is, the family $\{U(\varphi, \varepsilon): \varphi \in B, \varepsilon > 0\}$ is the open base, where $U(\varphi, \varepsilon) = \{\psi \in B: |\varphi - \psi| < \varepsilon\}$. Generally, B is a pseudo-metric space for this topology, and hence it may not be a Hausdorff space. The