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ON INTERTWINING DILATIONS. IV

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Abstract. We give a generalization of the theorems of the existence (see [9]) and the uniqueness (see [3]) of the contractive intertwining dilations in the presence of some representations of a C^*-algebra.

1. Let H_j $(j = 1, 2)$ be some (complex) Hilbert spaces and let $\mathscr{L}(H_1, H_2)$ denote the set of all (linear bounded) operators from H_1 into H_2. For a Hilbert space H, $\mathscr{L}(H)$ will stand for $\mathscr{L}(H, H)$. If $T \in \mathscr{L}(H_1, H_2)$ is a contraction, then we denote $D_T = (I - T^*T)^{1/2}$ and $\mathscr{D}_T = D_T(H_1)^-$. For a contraction $T \in \mathscr{L}(H)$, $U \in \mathscr{L}(K)$ will be the minimal isometric dilation of T; in other words:

$$K = H \oplus \mathscr{D}_T \oplus \mathscr{D}_T \oplus \cdots$$

and

$$U = \begin{pmatrix} T & 0 & 0 & \cdots \\ D_T & 0 & 0 & \cdots \\ 0 & I & 0 & \cdots \\ 0 & 0 & I & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

(For this and for any fact connected with the geometry of isometric dilations of contractions see [9], ch. I and II).

If $T_j \in \mathscr{L}(H_j)$ $(j = 1, 2)$ are two contractions, $I(T_1, T_2)$ will be the set of all operators $A \in \mathscr{L}(H_2, H_1)$ such that $T_1A = AT_2$. Let $U_j \in \mathscr{L}(K_j)$ be the minimal isometric dilation of T_j and P_j the (orthogonal) projection of K_j onto H_j $(j = 1, 2)$. For a contraction $A \in I(T_1, T_2)$, a *contractive intertwining dilation* ((T_1, T_2)-CID) of A will be a contraction $B \in I(U_1, U_2)$, such that $P_1B = AP_2$.

The existence of a (T_1, T_2)-CID for every contraction of $I(T_1, T_2)$ was proved by B. Sz.-Nagy and C. Foiaș in 1968 (see [9], ch. II, th. 2.3); recently T. Ando, Z. Ceaușescu and C. Foiaș proved in [3] that the uniqueness of the (T_1, T_2)-CID is equivalent to the fact that one of the factorizations $T_1 \cdot A$ or $A \cdot T_2$ be regular (in the sense of [9], ch. VII, §3). A generalization of this criterion is used in [6] for the uniqueness problem