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AN INVARIANT OF SYSTEMS IN THE ERGODIC THEORY

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1. Introduction. The following theorem is due to V. A. Rohlin and Ja. G. Sinai (cf. [3], [4]).

THEOREM. *Let* $(\Omega, \mathscr{F}, P)$ *be a separable complete probability space and* T *an invertible measure preserving transformation on* Ω. *Then there exists a sub-*σ*-field* $\mathscr{F}_0$ *such that*

$$\mathscr{F}_0 \subset T\mathscr{F}_0 , \quad \bigvee_{i\in Z} T^i\mathscr{F}_0 = \mathscr{F}, \quad \bigcap_{i\in Z} T^i\mathscr{F}_0 = \mathscr{P}(T) ,$$

where $\mathscr{P}(T)$ *denotes Pinsker's field and* Z *the set of all integers.*

Such a sub-σ-field $\mathscr{F}_0$ plays an important role in studies of mixing properties of transformations, but it is not uniquely determined by the transformation. The purpose of this note is to investigate the pair $(T, \mathscr{F}_0)$ which we call a system and to characterize the relation between the transformation and the sub-σ-field. Let $(S, \mathscr{G}_0)$ be another system defined on the same probability space. If there exists an invertible measure preserving transformation $R: \Omega \rightarrow \Omega$ such that $RT = SR$ and $R\mathscr{F}_0 = \mathscr{G}_0$, then these systems $(T, \mathscr{F}_0)$ and $(S, \mathscr{G}_0)$ are said to be isomorphic or more precisely, system-isomorphic. Our problem is to find a metric invariant of the system under system-isomorphy. We shall give an invariant employing a result due to J. de Sam Lazaro and P. A. Meyer [5], but this invariant is of "spectral" nature and is not complete with respect to the system-isomorphy, so it remains still a problem to find an invariant of "spatial" character.

In §§2 and 3 we shall construct two representations by applying the method given in [5], and further we shall make the theorem more precise so as to show the uniqueness of the representation. Then, this theorem determines the required invariant which we call the multiplicity of the system.

In §4 we shall show that the multiplicity of a Bernoulli system is equal to the dimension of the space of all squarely integrable functions which have zero expectations and are measurable with respect to the independent generator of this system.