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ON THE MULTIPLICITIES OF THE SPECTRUM FOR QUASI-CLASSICAL MECHANICS ON SPHERES

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0. Introduction. A. Weinstein [8] presented a quasi-classical calculation of the energy spectrum for a free particle moving on a sphere of constant curvature in any dimension. He showed that the quasi-classical spectrum resembles quite closely, in terms of both eigenvalues and multiplicities, the spectrum of the quantum hamiltonian $\Delta/2$. For the case of d-sphere S^d of constant sectional curvature one, his result is as follows: The quasi-classical eigenvalues are

$$\lambda_n = \frac{1}{2}\left(n + \frac{d-1}{2}\right)^2 \qquad \left(n > \frac{d-1}{2}\right),$$

and the multiplicity of λ_n is

$$m(\lambda_n) = \begin{cases} 2\begin{pmatrix} n + \dfrac{d-3}{2} \\ d-1 \end{pmatrix} & d \text{ odd}, \\ 2\begin{pmatrix} n + \dfrac{d-2}{2} \\ d-1 \end{pmatrix} & d \text{ even}. \end{cases}$$

Note that the counting starts with $n = (d+1)/2(d \text{ odd})$ or $n = d/2(d \text{ even})$. It is well-known that the eigenvalues of the quantum hamiltonian $\Delta/2$ on S^d are

$$\mu_n = \frac{1}{2}n(n+d-1) = \frac{1}{2}\left(n + \frac{d-1}{2}\right)^2 - \frac{(d-1)^2}{8},$$

and the multiplicity of μ_n is

$$m(\mu_n) = \frac{2n+d-1}{n}\begin{pmatrix} n+d-2 \\ n-1 \end{pmatrix},$$

where the counting starts with $n = 0$. See Berger-Gauduchon-Mazet [2].

In this note, we will present a slightly modified calculation of the quasi-classical energy spectrum for a free particle moving on S^d and