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COMPACTIFICATIONS OF THE MODULI SPACES OF HYPERELLIPTIC SURFACES

HIROYASU TSUCHIHASHI

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Hyperelliptic surfaces are classified into seven types [1]. In this paper we aim at constructing the moduli space of each type in the sense described in Mumford [8] and its compactification in some sense.

In Section 1, we study the structure of hyperelliptic surfaces and, by considering hyperelliptic surfaces with some base points, we get the fine moduli space M_i of each type as a quotient space of the upper half plane or the product of two copies of the upper half plane, over which Suwa [11] showed the existence of a family of hyperelliptic surfaces complete and effectively parametrized at each point.

In Section 2, we construct the compactification $\bar{M}_i$ of M_i. And as a preparation for Section 3, we describe the resolution of certain quotient singularities in terms of torus embeddings.

In Section 3, we describe "degenerate hyperelliptic surfaces" represented by the boundary points of $\bar{M}_i$.

1. We write the elements of $\boldsymbol{C}^2$ and $\boldsymbol{Z}^4$ as row vectors. Let Ω be a 2×2 matrix with coefficients in $\boldsymbol{C}$ of which the imaginary part is positive definite $\mathrm{Im}(\Omega) > 0$. Then by $A(\Omega)$ we denote the complex torus of dimension 2 with the period matrix Ω, i.e.,

$$A(\Omega) = \boldsymbol{C}^2/\boldsymbol{Z}^4\begin{pmatrix}\Omega\\ I\end{pmatrix}.$$

Let $[x, y]$ denote the point of $A(\Omega)$ which is the image of (x, y) in $\boldsymbol{C}^2$.

We denote the upper half plane by $\mathfrak{H}$, and for any element τ of $\mathfrak{H}$, we denote by $E(\tau)$ the elliptic curve with the periods 1 and τ, i.e.,

$$E(\tau) = \boldsymbol{C}/(\boldsymbol{Z}\tau + \boldsymbol{Z}).$$

We identify the elliptic curve E with its group $\mathrm{Aut}(E)^0$ of translations, and let $[x]$ denote the point of $E(\tau)$ which is the image of x in $\boldsymbol{C}$.

DEFINITION. By a hyperelliptic surface we mean an elliptic bundle over an elliptic curve whose total space has the first Betti number $b_1 = 2$.

THEOREM 1 [11]. *Hyperelliptic surfaces are topologically classified*