Tôhoku Math. Journ.
31 (1979), 309-318.

SCHWARZIAN DERIVATIVES OF SOME CONFORMAL MAPPINGS

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(Received August 2, 1978, revised December 12, 1978)

1. Introduction. As is well known, Nehari [5] proved the following theorem.

If f is a univalent meromorphic function defined in the unit disc, then

$$\sup_{|z|<1} |[f](z)|(1-|z|^2)^2 \leqq 6 , \tag{1}$$

where [f] is the Schwarzian derivative of f.

In this note, we are concerned with the case where the equality in (1) holds.

It is also well known that the Schwarzian derivatives of conformal mappings of the unit disc onto circular polygons are certain rational functions (see, for example, Goluzin [2]). First, by using such conformal mappings, we show that there exists a univalent meromorphic function for which the equality in (1) holds and whose Schwarzian derivative lies on the boundary of the Teichmüller space for a cyclic Fuchsian group. We also give a necessary condition in order that the equality in (1) holds and give an application of it.

The author is indebted to the referee for pointing out some errors in the original version of this note.

2. Notations and definitions. Let D be a simply connected domain in the extended complex plane $\hat{C}$ with more than one boundary point and let ρ_D be the Poincaré density of D, for example, $\rho_D(z) = (1-|z|^2)^{-1}$ if D is the unit disc. For a function ϕ holomorphic in D we introduce the norm

$$\|\phi\|_D = \sup_{z\in D} |\phi(z)|\rho_D(z)^{-2} .$$

We denote by $B_2(D, 1)$ the Banach space consisting of all the holomorphic functions ϕ in D which satisfy $\|\phi\|_D < \infty$.

For a locally univalent meromorphic function f in D, let $[f]$ be the Schwarzian derivative of f, that is,