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UMBILICS OF CONFORMALLY FLAT SUBMANIFOLDS

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0. Introduction. For $n \geqq 4$, let M be an n-dimensional conformally flat submanifold of the $(n+p)$-dimensional Euclidean space E^{n+p}. Recently under the assumption that M has the positive sectional curvature and $p \leqq n-3$, Sekizawa [3] proved that M contains an open subset on which there exists an involutive distribution of dimension $\geqq n-p$ such that each leaf of this distribution is totally umbilic in M and in E^{n+p}. In this note we show that the result of Sekizawa remains true without the assumption that the sectional curvature is positive.

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1. Statement of results. For $n \geqq 4$, let M be an n-dimensional conformally flat submanifold of the $(n+p)$-dimensional Euclidean space E^{n+p}. We denote the induced Riemannian metric on M by $\langle\ ,\ \rangle$, the Riemannian connection by ∇, the Ricci tensor by Ric, the scalar curvature by S, and the second fundamental form by α. The symmetric tensor Ψ is defined by

$$\Psi(X, Y) = [\mathrm{Ric}\,(X, Y) - \langle X, Y\rangle S/2(n-1)]/(n-2)$$

for $X, Y \in T_xM$. We now recall the notion of umbilic subspace of T_xM introduced in [3]. A subspace V of T_xM is said to be umbilic if $\dim V \geqq 2$ and $\alpha(X, X) = \alpha(Y, Y)$ for all unit vectors X and Y in V. Then our first result is the following.

PROPOSITION 1. *For $n \geqq 4$, let M be an n-dimensional conformally flat submanifold of the $(n+p)$-dimensional Euclidean space E^{n+p}. If $p \leqq n-3$ and $\mathscr{U}_x$ is the set of all vectors $X \in T_xM$ such that $\|\alpha(X, X)\|^2 = 2\|X\|^2\Psi(X, X)$, then*

(a) *$\mathscr{U}_x$ is the largest umbilic subspace of T_xM, and $\dim \mathscr{U}_x \geqq n-p$.*

(b) *For each unit vector $X \in \mathscr{U}_x$, the subspace $\{Y \in T_xM: \alpha(Y, Z) = \langle Y, Z\rangle\alpha(X, X)$ for all $Z \in T_xM\}$ is equal to $\mathscr{U}_x$.*

Let $p \leqq n-3$. Then by Proposition 1 we can define a distribution $\mathscr{U}$ by $M \ni x \mapsto \mathscr{U}_x$. We call $\mathscr{U}$ the umbilic distribution. The umbilic