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# THE VIRTUAL SINGULARITY THEOREM AND THE LOGARITHMIC BIGENUS THEOREM

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**Introduction.** When we study non-singular algebraic varieties $V$ defined over $\boldsymbol{C}$ the field of complex numbers, it is very important to know the *logarithmic Kodaira dimension* $\bar{\kappa}(V)$ of them $V$. In order to compute $\bar{\kappa}(V)$ of a non-singular algebraic variety $V$, we have to find a complete non-singular algebraic variety $\bar{V}^*$ and a divisor $D^\#$ with normal crossings on $\bar{V}^*$, such that $V = \bar{V}^* - D^\#$. Then by definition, $\bar{\kappa}(V) = \kappa(K(\bar{V}^*) + D^\#, \bar{V}^*)$. Here $\kappa(X, \bar{V})$ denotes the *X-dimension* of $\bar{V}$ (see [1]).

Occasionally, $V$ is given as a complement of a reduced divisor $D$ on a complete non-singular algebraic variety $\bar{V}$. In practice, it is very laborious to transform $D$ into $D^\#$ with normal crossings by a finite succession of blowing ups with non-singular centers. However, in general,

$$\bar{\kappa}(V) \leqq \kappa(K(\bar{V}) + D,\ \bar{V}) .$$

In many examples, we observe that the equality above holds actually. In such a case, we say that *the virtual singularity theorem holds for the pair* $(\bar{V}, D)$. For example, when $D$ has only normal crossings, the virtual singularity theorem holds by definition. If $\kappa(\bar{V}) \geqq 0$, the virtual singularity theorem holds with any effective divisor $D$. In this case, however, the strong virtual singularity theorem will be proved in Theorem 1. Moreover, even if $\bar{V}$ is a non-singular non-rational ruled surface, we can prove the virtual singularity theorem for $(\bar{V}, D)$ in Theorem 2.

On the other hand, when $\bar{V}$ is a rational surface (which is always assumed to be non-singular), the virtual singularity theorem does not hold in general. But even in this case, if $D$ has very bad singularities, we have the virtual singularity theorem (Theorem 4). This is a generalization of a theorem of Wakabayashi [10].

THEOREM (Wakabayashi). *Let $C$ be an irreducible curve of degree $d$ in $\boldsymbol{P}^2$.*

(1) *If $C$ is not rational and $d \geqq 4$, or*

(2) *if $C$ is a rational curve which has at least two singular*