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DYNAMIC BEHAVIOR FROM BIFURCATION EQUATIONS

Dedicated to Professor Taro Yoshizawa on his sixtieth birthday

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Abstract. Necessary and sufficient conditions for existence of small periodic solutions of some evolution equations can be obtained by the Liapunov-Schmidt method. In a neighborhood of zero, this gives a function (the bifurcation function) to each zero of which corresponds a periodic solution of the original equations. If this function is scalar, we show that its sign between the zeros gives the complete description of the stability properties of the periodic solutions.

1. Introduction. For the determination of solutions of an equation near a given solution, the method of Liapunov-Schmidt is very effective and has been applied to boundary value problems for ordinary, partial and functional differential equations, the problem of Hopf bifurcation for such equations, as well as many other problems. For several problems, this method reduces the discussion to the zeros of a function, called the *bifurcation function*, from a neighborhood of zero in one finite dimensional space to another finite dimensional space. The zeros of this function correspond to solutions of the original problem near the given solution, and conversely. Thus, the bifurcation function is a precise quantitative measure of the number of solutions of the original problem.

If the original problem corresponds to an evolutionary equation, one also must determine the stability properties of these solutions. It is the purpose of this paper to show that the bifurcation function may also carry the qualitative and quantitative dynamic behavior of the original problem. More precisely, consider the problem of the existence of 2π-periodic solutions of 2π-periodic ordinary, parabolic or retarded functional differential equations for which the linear part of the unperturbed equation has one zero eigenvalue and the remaining ones with

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