Tôhoku Math. Journ.
33 (1981), 127-131.

ON INTERTWINING BY AN OPERATOR HAVING A DENSE RANGE

EITOKU GOYA AND TEISHIRÔ SAITÔ

(Received February 28, 1980)

1. Throughout the paper, by an operator we mean a bounded linear transformation acting on a Hilbert space H. The algebra of all operators on H is denoted by $B(H)$.

We formulate an algebraic version of generalized Putnam-Fuglede theorem [3; Theorem 1], and we show that a paranormal contraction T is unitary, if S is a coisometry, if W is an operator having a dense range and if $TW = WS$. This is a generalization of a result due to Okubo [1].

Let $T \in B(H)$. T is *hyponormal* (resp. *cohyponormal*) if $T^*T - TT^* \geqq 0$ (resp. $TT^* - T^*T \geqq 0$). T is *dominant* if range $(T - \lambda) \subset$ range $(T - \lambda)^*$ for all $\lambda \in \sigma(T)$, the spectrum of T. This condition is equivalent to the existence of a constant M_λ for each $\lambda \in \sigma(T)$ such that

$$\|(T - \lambda)^* x\| \leqq M_\lambda \|(T - \lambda)x\|$$

for all $x \in H$. Thus every hyponormal operator is dominant. T is *paranormal* if

$$\|Tx\|^2 \leqq \|T^2 x\| \|x\|$$

for all $x \in H$.

2. The following theorem is a version of [3; Theorem 1]. The proof of [3] applies to this version. We include it for completeness.

THEOREM 1. *Let T, S, and $W \in B(H)$, where W has a dense range. Assume that $TW = WS$ and $T^*W = WS^*$. Then*

(i) *T is hyponormal (resp. cohyponormal), if so is S.*

(ii) *T is isometric (resp. coisometric), if so is S. In particular, T is unitary, if so is S.*

(iii) *T is normal, if so is S.*

PROOF. Let $W^* = V^*B$ be the polar decomposition of W^*. Since W has a dense range, W^* is injective. Thus $B^2 = WW^*$ is injective, and V is coisometric. From equations $TW = WS$ and $T^*W = WS^*$, we have