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HOLOMORPHIC FAMILIES OF RIEMANN SURFACES AND TEICHMÜLLER SPACES III

Bimeromorphic embedding of algebraic surfaces into projective spaces by automorphic forms

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Introduction. In this paper, as an application of the results in [4] and [5], we will deal with the bimeromorphic embedding of algebraic surfaces into projective spaces by automorphic forms.

Let X be a two-dimensional, irreducible, non-singular projective algebraic variety over $\boldsymbol{C}$. There exist a non-empty Zariski open subset $\mathscr{S}$ of X, a Riemann surface R of finite type and a holomorphic mapping $\pi: \mathscr{S} \to R$ so that the triple $(\mathscr{S}, \pi, R)$ is a holomorphic family of Riemann surfaces of type (g, n) with $2g-2+n>0$. We may assume that the universal covering space of R is the unit disc. Then the universal covering space $\tilde{\mathscr{D}}$ of $\mathscr{S}$ is a bounded Bergman domain in $\boldsymbol{C}^2$. Let $\tilde{\mathscr{G}}$ be the covering transformation group of the universal covering $\tilde{\Pi}: \tilde{\mathscr{D}} \to \mathscr{S}$. A holomorphic function f is called an automorphic form of weight q on $\tilde{\mathscr{D}}$ for $\tilde{\mathscr{G}}$, if

$$f(T(x)) = f(x)[J_T(x)]^{-q}$$

for all $T \in \tilde{\mathscr{G}}$ and $x \in \tilde{\mathscr{D}}$, where q is an integer and $J_T(x)$ is the Jacobian of T at x. We also say that f is a q-form for $\tilde{\mathscr{G}}$. We assume $q \geqq 2$ throughout this paper.

Our problem is stated as follows: *Can we construct many automorphic q-forms $f_0, \cdots, f_N$ for $\tilde{\mathscr{G}}$ in such a way that $F = (f_0, \cdots, f_N)$ induces a bimeromorphic embedding of X into the N-dimensional complex projective space $\boldsymbol{P}_N(\boldsymbol{C})$?* This problem is solved affirmatively in §8.

At the beginning, in §1, we recall the main results in [4] and [5]. In §2, we construct a domain $\mathscr{D}$ and a discrete subgroup $\mathscr{G}$ of the anålytic automorphism group of $\mathscr{D}$ so that our problem for $\tilde{\mathscr{D}}$ and $\tilde{\mathscr{G}}$ can be reduced to that for $\mathscr{D}$ and $\mathscr{G}$. §3 is devoted to constructing some auxiliary domains, which will be used in §7. In §4, we define the behaviour of automorphic forms for $\mathscr{G}$ near boundary points and, in §5, we recall some well-known results on the Poincaré metric and the