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DECOMPOSITION OF UNBOUNDED DERIVATIONS IN OPERATOR ALGEBRAS

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1. Introduction. The theory of unbounded derivations in operator algebras has been recently investigated by many authors (see for complete references, [10]), since the infinitesimal generators of the one-parameter groups of automorphisms in quantum dynamical systems are in general unbounded derivations. There are many examples of derivations which are not generators of dynamical systems and hence it may be important to study the property of unbounded derivations in C^*-algebras. Since a derivation in a C^*-algebra is extended to one in its enveloping von Neumann algebra, we shall mainly study derivations in von Neumann algebras.

In this paper we show that every (unbounded) $*$-derivation in a von Neumann algebra is decomposed into the sum of the normal part and the singular part, by using an algebra on an indefinite inner product space which is induced by the derivation.

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2. Preliminary results. We begin this section by giving the definition of derivations and introducing some notations.

By a *derivation* in a C^*-algebra $\mathfrak{A}$ (resp. a von Neumann algebra $\mathfrak{M}$), we mean a linear mapping δ of the domain $\mathscr{D}(\delta)$, which is a norm-dense (resp. σ-weakly dense) $*$-subalgebra of $\mathfrak{A}$ (resp. $\mathfrak{M}$), into $\mathfrak{A}$ (resp. $\mathfrak{M}$) such that

$$\delta(ab) = \delta(a)b + a\delta(b)$$

for each a, b in $\mathscr{D}(\delta)$. A derivation δ is called a $*$-derivation if $\delta(a^*) = \delta(a)^*$ holds for each a in $\mathscr{D}(\delta)$. Since every derivation can be expressed in the form $\delta_1 + i\delta_2$, where δ_1 and δ_2 are $*$-derivations. We shall only discuss $*$-derivations. It is well known that a derivation δ in a C^*-algebra $\mathfrak{A}$ with $\mathscr{D}(\delta) = \mathfrak{A}$ is necessarily norm-continuous and is also extended to a σ-weakly continuous derivation on the enveloping von Neumann algebra $\mathfrak{A}^{**}$. Let δ be a $*$-derivation in a C^*-algebra $\mathfrak{A}$ and let π be a