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THE DEFORMATION BEHAVIOUR OF THE KODAIRA DIMENSION OF ALGEBRAIC MANIFOLDS

(WITH AN APPENDIX BY K. UENO)

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0. **Introduction.** We consider the following:

PROBLEM. *Is the Kodaira dimension of algebraic manifolds invariant under deformation?*

For curves and surfaces, the answer is affirmative. In the former case the result is clear. In the latter case it was proved by Iitaka [I_2] using the classification of surfaces, whereas for non-algebraic complex manifolds of dimension three, Nakamura [N] produced a counterexample to this problem.

On the other hand, Lieberman-Sernesi [LS] introduced a notion of the relative Kodaira dimension $\kappa(X/Y)$ for a family $f: X \to Y$ of algebraic manifolds, and proved that the Kodaira dimension κ of fibers over a countable intersection of Zariski open sets of Y is equal to $\kappa(X/Y)$ and κ of other fibers are greater than $\kappa(X/Y)$. Using this notion, we formulate our problem in the following way.

CONJECTURE $DF_{n,k}$. *Let $f: X \to Y$ be a family of n-dimensional algebraic manifolds with $\kappa(X/Y) = k$. Then for any fiber $X_y = f^{-1}(y)$ $(y \in Y)$, we have $\kappa(X_y) = k$.*

Note that $DF_{n,n}$ is true by Lieberman-Sernesi's theorem. If all the Conjectures $DF_{n,-\infty}$, $DF_{n,0}$, $\cdots$, $DF_{n,n-1}$ are true, then the deformation invariance of the Kodaira dimension in the algebraic case will be settled.

In this paper, we study Conjecture $DF_{n,k}$ for $1 \leqq k \leqq n-1$. First we describe the geometric structure of every fiber of such a family as follows:

THEOREM I. *Let $f: X \to Y$ be a family of n-dimensional algebraic manifolds with $1 \leqq \kappa(X/Y) \leqq n-1$. Then for any $y \in Y$, the fiber X_y has the following property: There exist a nonsingular model X_y^* of X_y, a variety T and a fiber space $\psi: X_y^* \to T$ such that*

(1) $\dim T = \kappa(X/Y)$

(2) *There is an open set T' of T such that for any $t \in T'$, the*