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ON THE ANALYTICITY OF THE SOLUTIONS OF THE NAVIER-STOKES EQUATIONS

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1. Introduction. Consider the Navier-Stokes equation:

$$(1.1)\qquad \begin{cases} D_t u - \Delta u + \nabla p = f - \operatorname{div} N(u) & \text{in} \quad D^+ \times (0, T)\,, \\ \operatorname{div} u = 0 & \text{in} \quad D^+ \times (0, T)\,, \\ u|_{t=0} = u_0 \quad (\operatorname{div} u_0 = 0)\,, \quad u|_{x_3=0} = 0\,. \end{cases}$$

Here $N(u) = \{u_j u_k\}_{(j,k=1,2,3)}$ and

$$\operatorname{div} N(u) = \begin{pmatrix} \operatorname{div} N_1(u) \\ \operatorname{div} N_2(u) \\ \operatorname{div} N_3(u) \end{pmatrix}, \qquad N_j(u) = \begin{pmatrix} u_1 u_j \\ u_2 u_j \\ u_3 u_j \end{pmatrix}.$$

The set D is a neighborhood of the origin in the three dimensional Euclidean space E_3 and $D^+ = D \cap E_3^+$ with $E_3^+ = \{x = (x_1, x_2, x_3) \in E_3; x_3 > 0\}$. Let Ω and $\mathscr{D}$ be some complex neighborhoods of $(0, T)$ and D, respectively. Let $C^{r,r/2}(D^+ \times \Omega)$ be a weighted Hölder space. Now our result is as follows:

THEOREM 1.1. *Let f and u_0 be analytically extended from $D \times (0, T)$ and D to $\mathscr{D} \times \Omega$ and $\mathscr{D}$, respectively. Let $u \in C^{2+\mu,(2+\mu)/2}(D^+ \times \Omega)$ and $p \in C^{1+\mu,(1+\mu)/2}(D^+ \times \Omega)$ satisfy the equation (1.1) which are analytic in $\omega \in \Omega$ for each $x \in D^+$ $(0 < \mu < 1)$. Then $u(x, t)$ and $p(x, t)$ are analytic near $(0, t_0)$ for any t_0 $(0 < t_0 < T)$.*

The analyticity of the solutions was proved in Kahane [3] and Masuda [7], but they only proved the interior analyticity.

Many authors have proved the analyticity of the solutions of elliptic and parabolic equations, for example, Friedman [1], Morrey [8], etc. There are several methods to prove the analyticity. We will here use the method of Morrey. First, by Morrey [8], we shall show that there exists a complex analytic extension of the solution of the associated Stokes equation in a half space. Next, we will decompose the solution (u, p) of (1.1) into $u = u' + u''$, $p = p' + p''$, respectively. Here (u', p') is the solution of some integral equation and (u'', p'') is the solution of some Stokes equation. We will prove that they and their first spatial