Tôhoku Math. Journ.
33 (1981), 593-597.

ENTROPY AND ALMOST EVERYWHERE CONVERGENCE OF FOURIER SERIES

SHUICHI SATO

(Received May 26, 1981)

1. **Introduction.** Fefferman [3] has developed a theory of Fourier analysis using the notion of entropy. He showed in particular that entropy arguments are useful to investigate several problems in Fourier analysis which occur near L^1-space, e.g., the Zygmund class $L\log^+ L$.

In this note we shall prove that if the entropy of f in $[-\pi, \pi]$ is finite, then the partial sums of the Fourier series of f converge a.e. Our proof depends essentially on Carleson's famous theorem in [2]. On the other hand, we shall give a function whose entropy is finite but which does not belong to $L\log^+ L\log^+\log^+ L$ or the L^1-Dini class. This fact will be interesting if we recall Sjölin's theorem which implies that if $f\in L\log^+ L\log^+\log^+ L$, then the Fourier series of f converges a.e.

Our result will be extended to the Riesz-Bochner means of the Fourier series of a function of several variables. Let $Q=\{x=(x_1, \cdots, x_d)\mid -\pi< x_i\leqq \pi\}$ be the fundamental cube in $\boldsymbol{R}^d$. For a set $S\subset Q$ the entropy $E(S)$ of S is defined by

$$E(S)=\inf_{S\subset\cup Q_k}\sum_{k\geqq 1}|Q_k|\log|Q_k|^{-1}\ ,$$

where Q_k are subcubes of Q, and the entropy $J(f)$ of a nonnegative function f is defined by

$$J(f)=\int_0^\infty E(\{x\in Q\mid f(x)>\lambda\})d\lambda\ .$$

For these definitions and basic properties of $E(S)$ and $J(f)$ we refer to Fefferman [3]. Furthermore we define the L^1-Dini class as the class of functions which have the finite L^1-Dini norm $\|f\|_{D^1}$ defined by

$$\|f\|_{D^1}=\|f\|_{L^1}+\iint_{Q^2}\frac{|f(x)-f(y)|}{|x-y|^d}dxdy\ .$$

Fefferman [3] has proved that if f belongs to the L^1-Dini class, then $J(f)$ is finite, and if $J(f)$ is finite, then f is in the class $L\log^+ L(Q)$.

2. **Theorem.** Let $d\geqq 1$ and let f be an integrable function on Q. The spherical Riesz-Bochner mean of order δ of f is defined by