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CERTAIN DECOMPOSITIONS OF BMO-MARTINGALES

YASUNOBU SHIOTA

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1. **Introduction.** In [6] we considered a martingale version of the results in Coifman and Rochberg [1] under the condition that every martingale is continuous. This continuity condition made it possible to use the Varopoulos decomposition (see Varopoulos [7]) and to avoid some technical difficulties caused by jumps of sample paths. In this note, instead of the Varopoulos decomposition, we use the Herz-Lépingle representation of BMO-martingales (see Lemma 2 below), which, combined with the section theorem, enables us to remove the continuity condition.

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2. **Preliminaries.** Let $(\Omega, F, P; (F_t)_{t\in R^+})$ be a probability system which satisfies the usual conditions. We assume that the reader is familiar with the theory of general processes, especially the section theorem and the martingale theory. In the sequel T denotes the F_t-stopping time. Note that the constant C is not always the same in each occurrence.

DEFINITION 1. A uniformly integrable martingale $X = (X_t)$ is said to be a BMO-martingale if $\|X\|_{\mathrm{BMO}} = \sup_T \operatorname{ess.sup} E[|X_\infty - X_{T-}||F_T]$ is finite.

We denote by BMO the class of all BMO-martingales. BMO is a Banach space with the norm $\| \quad \|_{\mathrm{BMO}}$.

The following lemmas are well-known. For the proof, see Meyer [4] and [3] respectively.

LEMMA 1 (the inequality of John-Nirenberg's type). *Let X be a* BMO-*martingale. If $\alpha < 1/(8\|X\|_{\mathrm{BMO}})$, then $E[\exp \alpha|X_\infty - X_{T-}||F_T] < \infty$ a.s. for every T.*

LEMMA 2 (the Herz-Lépingle representation). *Let X be a* BMO-*martingale. Then there is a non-adapted process $B = (B_t)$ (not necessarily unique) such that* (a) $\int_0^\infty |dB_s| \leq C$ *for some constant C and* (b) $X_\infty = A_\infty$, *where A is the optional dual projection of B.*