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ESTIMATES FOR THE ASYMPTOTIC ORDER OF A GRÖTZSCH RING CONSTANT

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Abstract. Asymptotic approximations in terms of n are obtained for the constant $\log \lambda_n = \lim_{a\to 0} (\operatorname{mod} R_{G,n}(a) + \log a)$ associated with the Grötzsch extremal ring $R_{G,n}$ in euclidean n-space, $n \geqq 3$.

1. Definitions and notation. By a *ring* R is meant a domain in finite euclidean n-space whose complement consists of two components C_0 and C_1, where C_0 is bounded. The *conformal capacity* of R (cf. [11]) is

$$\operatorname{cap} R = \inf_{\varphi} \int_R |\nabla\varphi|^n d\omega ,$$

where ∇ denotes the gradient, and where the infimum is taken over all real-valued C^1 functions φ in R with boundary values 0 on ∂C_0 and 1 on ∂C_1. Then the *modulus* of the ring R is defined by

$$\operatorname{mod} R = (\sigma_{n-1}/\operatorname{cap} R)^{1/(n-1)} ,$$

where for each positive integer p we let σ_p denote the p-dimensional measure of the unit sphere

$$S^p = \left\{(x_1, \cdots, x_{p+1}) \colon \sum_{j=1}^{p+1} x_j^2 = 1\right\} .$$

Then

$$(1) \qquad \sigma_p = 2\pi^{(p+1)/2}\Gamma((p+1)/2)^{-1}$$

(cf. [9], [12]), where Γ denotes the classical Gamma function. For later reference we recall that

$$(2) \qquad \int_0^{\pi/2} \cos^p u du = \sigma_{p+1}/2\sigma_p$$

for each positive integer p (cf. [2]).

For $n \geqq 2$ and $0 < a < 1$ we let $R_{G,n} = R_{G,n}(a)$ denote the n-dimensional Grötzsch ring, that is, the ring whose complementary components are

$$C_0 = \{(x_1, \cdots, x_n) \colon 0 \leqq x_1 \leqq a,\ x_j = 0,\ 2 \leqq j \leqq n\}$$

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