Tôhoku Math. Journ.
34 (1982), 101-112.

ON THE STRUCTURE OF THE IDELE GROUPS OF ALGEBRAIC NUMBER FIELDS, II

KATSUYA MIYAKE

(Received April 18, 1981)

In this paper, we make further and more precise investigation into the idele groups of algebraic number fields than in our previous paper [4].

Here we state, as an example, a theorem obtained in §7 in a somehow weakened and simplified form, which, even so, includes the main result [4, Theorem 2] as a special case:

THEOREM. *Let L be a finite Galois extension of an algebraic number field F, and V an open subgroup of the idele group $L_A^\times$ of L which contains $L^\times \cdot L_{\infty+}^\times$ and satisfies* $(*)$ $V^\sigma = V$ *for any* $\sigma \in \mathrm{Gal}(L/F)$ *and* $(**)$ $L_A^\times = F_A^\times \cdot V \cdot N_{L/F}^{-1}(F^\times)$. *Then* $F_A^\times \cap V = F_A^\times \cap V \cdot N_{L/F}^{-1}(F^\times)$.

Our basic tool is Terada's theorem on transfers of a finite group, which is generalized in §4.

In the final section, we point out a few results on capitulation of ideals easily derived from what we obtain the previous sections.

1. Preliminaries. For an algebraic number field F, we denote the ring of adeles of F by F_A, and the idele group by $F_A^\times$. Let $F_A^\times = F_f^\times \cdot F_\infty^\times$ be the decomposition of $F_A^\times$ into the product of its non-Archimedian part $F_f^\times$ and its Archimedian part $F_\infty^\times$. The connected component of the unity of $F_\infty^\times$ is denoted by $F_{\infty+}^\times$, and the topological closure of $F^\times \cdot F_{\infty+}^\times$ in $F_A^\times$ by $F^\#$. Let F_{ab} be the maximal abelian extension of F in the algebraic closure of F. The Artin map $[\cdot, F]: F_A^\times \to \mathrm{Gal}(F_{\mathrm{ab}}/F)$ of class field theory is an open, continuous and surjective homomorphism, whose kernel is $F^\#$.

Let K be a finite abelian extension of F, and put $\mathfrak{g} = \mathrm{Gal}(K/F)$. Then $\mathfrak{g}$ acts on $K_A^\times$ naturally. Let $G_{K,F}$ be the Weil group of the extension K over F. This is the extension of the idele class group $K_A^\times/K^\times$ by $\mathfrak{g}$, which corresponds to the canonical class $\xi_{K/F}$ in the cohomology group $H^2(\mathfrak{g}, K_A^\times/K^\times)$. (See Weil [7] and Hochschild and Nakayama [2], or Iyanaga [3, Ch. 5, §6].) There exists a surjective homomorphism $\phi_{K,F}: G_{K,F} \to \mathrm{Gal}(K_{\mathrm{ab}}/F)$ whose kernel is $K^\#/K^\times$ and whose restriction to the subgroup $K_A^\times/K^\times$ coincides with the homomorphism induced by the