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A PROBLEM ON ORDINARY FINE TOPOLOGY AND NORMAL FUNCTION

To the memory of Lu San Chen

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Abstract. Earlier in 1961, Doob proved that if $f(z)$ is a normal function in a disk, then every angular cluster value at a boundary point is also a fine cluster value at the point. He then asked whether or not the converse of this theorem is true. In this paper, we answer this question in the negative sense with respect to the ordinary fine topology of Brelot.

1. **Introduction.** Let $D(|z| < 1)$ and $C(|z| = 1)$ be the unit disk and circle respectively. Let $f(z)$ be a function defined in D. We say that the function f has an angular cluster value v at a boundary point $p \in C$, if there is an angle $A(p)$ lying in D with one vertex at p and a sequence $\{p_n\}$ of points $p_n \in A(p)$ such that

$$\lim_{n\to\infty} p_n = p \quad \text{and} \quad \lim_{n\to\infty} f(p_n) = v .$$

We shall now introduce the notion of fine topology in the sense of Brelot [2, p. 327]. Let E be a set and p a point. We say that E is thin at the point p, if either p is not a limit point of E or there exists a superharmonic function $s(z)$ such that

$$s(p) < \liminf_{z\to p} s(z) , \quad \text{where} \quad z \in E - p .$$

The first case is trivial and therefore only the second case will be considered in the sequel.

With the notion of thinness, we can now follow Doob [3 or 4] to define the fine cluster value. We say that the function f has a fine cluster value v at a point $p \in C$, if there is a set $T \subset D$ which is not thin at p and

$$\lim_{z\to p} f(z) = v , \quad \text{where} \quad z \in T .$$

In this case, the point p is called a fine limit point of the set T.

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