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EXTENSIONS OF DERIVATIONS AND AUTOMORPHISMS FROM C^*-ALGEBRAS TO THEIR INJECTIVE ENVELOPES

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1. Introduction and preliminaries. Quite recently, the first author showed that any unital C^*-algebra A has a unique injective envelope $I(A)$ which indeed is an AW^*-algebra and contains the regular monotone completion $\bar{A}$ of A as an AW^*-subalgebra. The injective envelope $I(A)$ (resp. the regular monotone completion $\bar{A}$) reflects closely the structure of A; e.g., any $*$-automorphism of A is extended to a unique $*$-automorphism of $I(A)$ (resp. $\bar{A}$) ([6]).

AW^*-algebras are more tractable than the general C^*-algebras. They have sufficiently many projections and are decomposed uniquely according to type. Moreover it is known that their derivations are inner ([10]).

On the other hand, $I(A)$ is an AW^*-factor if and only if A is prime, and in most cases $I(A)$ becomes a non-W^*, AW^*-algebra. To such an algebra the spatial theory of W^*-algebras cannot be applicable and to study it seems to be very interesting.

In this paper we shall consider the following questions: Whether can each derivation on a C^*-algebra be extended to a unique derivation on its injective envelope and whether can each automorphism (not necessarily $*$-preserving) of a C^*-algebra be extended to a unique automorphism of its injective envelope? The answers should be given *affirmatively* to both questions for a general C^*-algebra. As an application of the observation on derivations, we shall be able to introduce, for the general C^*-algebra A, the C^*-algebra $D(A)$, as a C^*-subalgebra of the regular monotone completion $\bar{A}$ of A (note that, if A is separable then $\bar{A}$ coincides with the regular σ-competion $\hat{A}$ of A [18] and hence $D(A)$ is a C^*-subalgebra of $\hat{A}$). This C^*-algebra $D(A)$ must coincide with Sakai's derived algebra $\mathscr{D}(A)$ ([14]) if A is factorial (see also Tomiyama [17]).

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