Tôhoku Math. Journ.
34 (1982), 587-629.

# HOLOMORPHIC STRUCTURES MODELED AFTER HYPERQUADRICS

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(Received April 20, 1982)

**1. Introduction.** In our joint paper with Inoue [7], we studied holomorphic affine connections and affine structures on complex manifolds and classified all compact complex surfaces admitting such structures. In [12] we studied holomorphic projective connections and projective structures and classified all compact complex surfaces admitting such structures. The one case left open in [12] has been solved recently ([13]). Both of our papers were partly based on Gunning's earlier work [4].

In the present paper we shall study holomorphic geometric structures modeled after a hyperquadric. Leaving the precise definitions of holomorphic $CO(n; C)$-structure and quadric structure to § 2, we shall explain them by the following diagram:

| *Model space* | *Infinitesimal structure* | *Local structure* |
|---|---|---|
| Affine space $C^n$ | Affine connection | Affine structure |
| Projective space $P_nC$ | Projective connection | Projective structure |
| Quadric $Q_n$ | $CO(n; C)$-structure | Quadric structure |

By a quadric $Q_n$ we mean a non-singular hyperquadric in $P_{n+1}C$; it is a holomorphic analogue of a sphere. A holomorphic $CO(n; C)$-structure may be considered as a holomorphic conformal connection, and a quadric structure as a flat holomorphic conformal structure.

In § 2, § 3 and § 4, we shall discuss general results valid for all dimension. In the subsequent sections we determine all compact complex surfaces admitting holomorphic $CO(2; C)$-structures and quadric structures. The 2-dimensional case is somewhat exceptional as in the case of conformal differential geometry. This is because a non-singular quadric $Q_2$ is isomorphic to $P_1C \times P_1C$, i.e., reducible. Hence, a holomorphic $CO(2; C)$-structure is equivalent (modulo passing to a double covering) to a splitting of the holomorphic tangent bundle into a direct sum of two holomorphic line subbundles, which in turn, is equivalent to a pair of mutually transversal holomorphic foliations of dimension 1. We take a

* Partially supported by NSF Grant MCS 79-02552.