Tôhoku Math. Journ.
35 (1983), 137-153.

A UNIFORMITY OF DISTRIBUTION OF G_Q IN G_A

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(Received August 13, 1982)

Introduction. 0-1. Let G be a connected and semisimple linear algebraic group defined over $\boldsymbol{Q}$. Denote by G_Q and G_A the group of $\boldsymbol{Q}$-rational points of G and the adele group of G, respectively. We identify G_Q with a subgroup of G_A in a natural manner. Then G_Q is discrete in G_A and the quotient $G_Q \backslash G_A$ has a finite volume for a G_A-invariant measure.

In his paper [6], Kuga proposed the following problem:

How the set of points of G_Q is distributed in G_A?

He gave an answer for the case that G is a $\boldsymbol{Q}$-form of SL(2) of Q-rank 0. With the help of Kuga's basic idea ("Kuga's criterion", see Proposition 1) and a deep result of representation theory due to Howe and Moore [5], the present author [9] showed a uniformity of distribution of G_Q in G_A with respect to a Haar measure dg on G_A when G is simply connected, absolutely almost simple and furthermore has $\boldsymbol{Q}$-rank zero. Roughly speaking, we showed that, for a relatively compact open subset X of G_A, the main term of the number $|X \cap G_Q|$ is equal to $\int_X dg$, if $\int_X dg$ is sufficiently large. Here we normalize the Haar measure dg on G_A by $\int_{G_Q \backslash G_A} dg = 1$. (In fact, we must impose some additional conditions on X. For detail, see Theorem.)

The object of the present paper is to show that the above result is also available even if G has $\boldsymbol{Q}$-rank greater than zero.

0-2. To explain our result more precisely, denote by G_f (resp. G_∞) the finite (resp. infinite) part of G_A; $G_f = \prod'_p G_{Q_p}$ (the restricted direct product), $G_\infty = G_R$. Then we have $G_A = G_f \cdot G_\infty$. For a finite set $\mathscr{S}$ of finite places of $\boldsymbol{Q}$, put $G_f(\mathscr{S}) = \prod_{p \in \mathscr{S}} G_{Q_p} \times \prod_{p \notin \mathscr{S}} G_{Z_p}$, which is an open subgroup of G_f.

Consider a sequence $\{X_j\}_{j=1}^\infty$ of relatively compact open subsets of G_A. A sequence $\{X_j\}_{j=1}^\infty$ is said to be *of Hecke type* if the following two conditions (0.1)-(0.2) are satisfied:

(0.1) Each X_j has the form $S(j) \times U$, where $S(j)$ is an open compact subset of $G_f(\mathscr{S})$ for a fixed finite set $\mathscr{S}$ of finite places of $\boldsymbol{Q}$ and U is a fixed relatively compact domain in G_∞.