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A RELATION BETWEEN THE TOTAL CURVATURE AND THE MEASURE OF RAYS, II

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1. Introduction. The total curvature of complete, noncompact, connected and oriented Riemannian 2-manifold M is defined to be an improper integral $\int_M Gdv$ of the Gaussian curvature G of M with respect to the Riemannian volume dv over M. It is well known that the total curvature of such an M is not a topological invariant but it depends on the choice of the Riemannian structure. The pioneering work of Cohn-Vossen on the total curvature states that if M is finitely connected and if M admits the total curvature, then $\int_M Gdv \leqq 2\pi\chi(M)$, where $\chi(M)$ is the Euler characteristic of M (see [2, Satz 6]). It is interesting to investigate a geometric influence of the total curvature on the Riemannian structure of M which defines it. The first attempt in this direction of the work was made by Maeda in [5], [6] and [7]. He investigated some relations between the measure of rays emanating from a fixed point and the total curvature of a complete Riemannian manifold homeomorphic to $\boldsymbol{R}^2$ whose Gaussian curvature is nonnegative everywhere.

From completeness and compactness of M it follows that through every point p on M there passes at least a ray $\gamma: [0, \infty) \to M$, where a ray is by definition a unit speed geodesic such that any subarc of it is a unique minimizing geodesic between the endpoints. Here all geodesics are parametrized by arc length unless otherwise mentioned. For a point p on M let T_pM and S_pM be the tangent space to M at p and the unit circle of T_pM centered at the origin. S_pM is endowed with a natural Lebesgue measure induced by the Riemannian structure of M. Let $A(p)$ be the set of all unit vectors tangent to rays emanating from p. $A(p)$ is closed because a limit geodesic of a sequence of rays is again a ray. Thus we are interested in the measure of the set $A(p)$. In a recent paper, Maeda has proved the following:

THEOREM (Maeda [7]). *If M is a complete Riemannian manifold homeomorphic to $\boldsymbol{R}^2$ and if the Gaussian curvature G of M is nonnegative everywhere, then*