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THE MULTIPLICITY OF HELICES FOR A REGULARLY INCREASING SEQUENCE OF σ-FIELDS

Dedicated to Professor Tamotsu Tsuchikura on his sixtieth birthday

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1. Introduction. The notion of helices was introduced in the theory of measure-preserving transformations as an application of the martingale theory by J. de Sam Lazaro and P. A. Meyer [2]. The multiplicity of helices has been discussed by the author in the same manner as that of square-integrable martingales [4], [5]. In this paper, we determine the multiplicity of helices under some condition of the regularity on an increasing sequence of sub-σ-fields.

2. Preliminaries. Throughout this paper (Ω, F, P) denotes a complete separable probability space and T an automorphism of Ω, that is, a bimeasurable measure-preserving bijection. Let F_0 be a complete proper sub-σ-field of F such that

$$\text{(a)}\quad F_n \subset F_{n+1} \quad \text{for all} \quad n \in Z\,,$$

$$\text{(b)}\quad \bigvee_{n \in Z} F_n = F$$

where $Z = \{0, \pm 1, \pm 2, \cdots\}$ and F_n denotes the sub-σ-field $T^n F_0$. A pair (T, F_0) is called a system.

Let H denote the class of all squarely integrable real random variables with expectations 0, which is an infinite dimensional Hilbert space under the ordinary inner product, and H_n the subspace of H consisting of all elements measurable with respect to F_n for each $n \in Z$.

DEFINITION 1. A sequence $X = (x_n)_{n \in Z}$ in H is called a helix of (T, F_0) if the following conditions are satisfied:

$$\text{(a)}\quad x_0 = 0\,,$$

$$\text{(b)}\quad x_n - x_{n-1} \in H_n \cap H_{n-1}^{\perp} \quad \text{for all} \quad n \in Z$$

where $\perp$ indicates the orthogonal complementation in H,

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