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THE SECOND COHOMOLOGY GROUPS OF THE GROUP OF UNITS OF A Z_p-EXTENSION

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Let p be a prime number. We denote by K_0 a finite algebraic number field, by K_∞ a Z_p-extension of K_0 and by K_n the cyclic subextension of degree p^n. Let E_∞ be the unit group of K_∞. Put $\Gamma_n = \mathrm{Gal}(K_n/K_0)$ and $\Gamma = \mathrm{Gal}(K_\infty/K_0)$. In connection with the Leopoldt conjecture and Greenberg conjecture, Iwasawa [5] posed the problem of studying the structure of $H^2(\Gamma, E_\infty)$. Let S_n be the set of prime ideals of K_n ramified in K_∞, and D_n be the p-Sylow subgroup of the ideal class group generated by the ideals $\prod \mathfrak{p}^\sigma$ for $\mathfrak{p} \in S_n$, where $\mathfrak{p}^\sigma$ runs through all different conjugates of $\mathfrak{p}$ over K_0. We consider the inductive limit D_∞ of D_n by means of the natural map. In this paper, we shall give a partial answer,

$$H^2(\Gamma, E_\infty) \cong (Q_p/Z_p)^{s_0 - r_p - 1}$$

where $r_p = \text{ess. rank}\, D_\infty$ and $s_0 = \sharp S_0$.

While preparing this paper, the author received the preprint by Iwasawa entitled "On cohomology groups of units for Z_p-extensions" in which he also obtains a similar result. (The paper has since appeared in [6].)

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1. We define the essential rank for a Z_p-module M, which is denoted by ess. rank M, as the dimension of $M \otimes_{Z_p} Q_p$. For a p-primary torsion abelian group, we also define the essential rank as that of the Pontrjagin dual.

LEMMA 1. *Let $\{M_n\}_{n \geqq 0}$ be a family of finite abelian p-groups with bounded p-ranks. For $m > n \geqq 0$, let $\varphi_{m,n}: M_n \to M_m$ (resp. $\psi_{m,n}: M_m \to M_n$) be homomorphisms giving rise to an inductive system $\{M_n, \varphi_{m,n}\}$ (resp. projective system $\{M_n, \psi_{m,n}\}$). If the orders of* $\mathrm{Ker}(\varphi_{m,n})$ *and* $\mathrm{Coker}(\psi_{m,n})$ *are bounded with respect to m and n, then we have* ess. rank ind lim$\{M_n, \varphi_{m,n}\}$ = ess. rank proj lim$\{M_n, \psi_{m,n}\}$.

PROOF. Let M_n^* be the dual abelian group of M_n and $\psi_{m,n}^*$ be the