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CODIMENSION ONE TOTALLY GEODESIC FOLIATIONS OF $\boldsymbol{H}^n$

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Introduction. The characterization of isometric immersions of codimension 1 between hyperbolic spaces was proposed by Nomizu in [7], where this problem is compared with the corresponding ones for space forms of positive and zero curvature.

In [4], Ferus classified the umbilic-free immersions $\boldsymbol{H}^n \to \boldsymbol{H}^{n+1}$ sharing a given relative nullity foliation T_0, which was determined by the arbitrary choice of an orthogonal trajectory. As the leaves of T_0 are hyperspaces (complete totally geodesic hypersurfaces) in $\boldsymbol{H}^n$, such a curve will be enough to determine T_0 uniquely but the converse is not true, for two orthogonal trajectories of a hyperspace foliation need not be related by a congruence (rigid motion of $\boldsymbol{H}^n$). This paper offers a classification of hyperspace foliations of $\boldsymbol{H}^n$, up to congruence, which includes non-smooth foliations too. Such generality is needed in the study of immersions that may have umbilics (see [1]).

Basic results of Riemannian geometry assumed here will be found in Kobayashi-Nomizu [6]. Several other facts, more specific of hyperbolic geometry, were less readily available at least in the form needed here. Sections 2 and 3 deal with this material. Preparation of those sections was made easier by [2]. The author wishes to thank K. Nomizu for suggesting the problem that originated this work and for a great deal of further assistance and advice.

1. Notation and Terminology.

We shall deal with smooth $(=C^\infty)$ manifolds endowed with linear connections. Given such a manifold H, a geodesic will be assumed to have the largest possible domain. Since H will almost always be (geodesically) complete, a geodesic will then be a *map*

$$\sigma: \boldsymbol{R} \to H$$

whose velocity vector is parallel. The *set* $\sigma(\boldsymbol{R})$ will be referred to as the *path of* σ. A (geodesic) segment is the image, under a geodesic, of a finite interval whereas a (geodesic) ray is the restriction of a geodesic to an interval of type $(-\infty, a]$ or $[a, \infty)$. Unless otherwise stated, a

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