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STABILITY OF CERTAIN MINIMAL SUBMANIFOLDS OF COMPACT HERMITIAN SYMMETRIC SPACES

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Introduction. In this paper we consider a compact totally real totally geodesic submanifold M of a Hermitian symmetric space $(\bar{M}, \bar{g})$ of compact type with $\dim M = \dim_C \bar{M}$, and study their classification and stability.

We shall show that *such a submanifold M is always a symmetric R-space* (cf. §1 for definition), *and these pairs $((\bar{M}, \bar{g}), M)$ correspond in one to one fashion to symmetric R-spaces.* Furthermore we shall prove that *M is stable in $(\bar{M}, \bar{g})$ as a minimal submanifold if and only if M is simply connected.*

Lawson-Simons [6] proved that a compact stable minimal submanifold of the complex projective n-space $P_n(C)$ endowed with the Kähler metric of constant holomorphic sectional curvature is always a complex submanifold. They showed also [6] that this is not true for a general Hermitian symmetric space of compact type, by giving an example of a compact stable minimal submanifold of $P_1(C) \times P_1(C)$ which is not a complex submanifold. The simply connected ones among our submanifolds include the example of Lawson-Simons and provide many examples with the same properties. For example, the quaternion Grassmann manifold $G_{p,q}(\boldsymbol{H})$ imbedded in the complex Grassmann manifold $G_{2p,2q}(C)$ is minimal and stable, but not a complex submanifold.

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1. Totally real totally geodesic submanifolds of compact Hermitian symmetric spaces.

In this section we shall classify compact totally real totally geodesic submanifolds M of a Hermitian symmetric space $(\bar{M}, \bar{g})$ of compact type with $\dim M = \dim_C \bar{M}$.

Let $(\bar{M}, \bar{g})$ be a Hermitian manifold. The inner product and the complex structure tensor on the tangent bundle $T\bar{M}$ are denoted by $\langle\ ,\ \rangle$ and J, respectively. A submanifold M of $\bar{M}$ is said to be *totally real* if $\langle JT_pM, T_pM\rangle = 0$ for each $p \in M$. A submanifold M is called a *real form*