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HOMOGENEOUS MINIMAL HYPERSURFACES IN THE UNIT SPHERES AND THE FIRST EIGENVALUES OF THEIR LAPLACIAN

HIDEO MUTO, YOSHIHIRO OHNITA AND HAJIME URAKAWA

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1. Introduction. The geometry of minimal closed submanifolds of the unit sphere is closely related to the eigenvalue problem of the Laplacian. In this paper, we study the first eigenvalue of the embedded minimal hypeasurfaces in the unit sphere.

Let (M, g) be an n-dimensional compact connected Riemannian manifold without boundary and Δ its (non-negative) Laplacian acting on C^∞-functions on M. Let $\{0 = \lambda_0 < \lambda_1 \leqq \lambda_2 \leqq \cdots \uparrow \infty\}$ be its spectrum. λ_1 is called the first eigenvalue of Δ. Let f be an isometric immersion of (M, g) into the N-dimensional standard unit sphere $S^N(1)$ of the Euclidean space $\boldsymbol{R}^{N+1}$ with the coordinate $(x^0, x^1, \cdots, x^N)$. Then it is known (cf. [13]) that the $N+1$ functions $x^i \circ f$ $(i = 0, 1, \cdots, N)$ on M are the eigenfunctions of Δ with the eigenvalue n if and only if $f(M)$ is minimal in $S^N(1)$. Therefore the first eigenvalue λ_1 of Δ of an n-dimensional minimally isometrically immersed Riemannian manifold (M, g) in $S^N(1)$ is not greater than n. In particular, for the great sphere $S^n(1)$ and the generalized Clifford torus $S^p(\sqrt{p/n}) \times S^q(\sqrt{q/n})$ $(p + q = n)$ of $S^{n+1}(1)$, the first eigenvalue λ_1 is just n. In this connection, Ogiue [10] posed the following problem:

PROBLEM (A). What kind of embedded minimal hypersurfaces of $S^{n+1}(1)$ have n as the first eigenvalue of its Laplacian?

Yau [18] posed independently a similar problem. In this paper, we consider a little more restricted problem:

PROBLEM (B). Is n the first eigenvalue of the Laplacian for the embedded homogeneous minimal hypersurfaces of $S^{n+1}(1)$?

In this paper we give a partial answer to the Problem (B) using the classification (cf. [5]) of homogeneous hypersurfaces of the unit sphere and the theory of spherical functions on a compact homogeneous space:

THEOREM. *Let (M, g) be an embedded homogeneous minimal hypersurface in the unit sphere which is diffeomorphic to one of the following*:

(i) $\mathrm{SO}(3)/\boldsymbol{Z}_2 \times \boldsymbol{Z}_2$,